Online Appendices for Political Regimes and Foreign Intervention

1 Online appendix A

This appendix contains the formal details underlying the discussion of the equilibrium regime typology in section 3 of the printed text.

1.1 Deriving the optimal tax structure under consolidated democracy and autocracy

In a consolidated democracy, the median voter (a worker) solves the following fiscal problem:

$$\max_{\tau_L,\tau_\pi} A\left(1-L\right) - \tau_L A\left(1-L\right)^2 + \tau_\pi \Pi.$$

It is clear that $\tau_L^{\mathcal{D}} = 0$ and $\tau_\pi^{\mathcal{D}} = \overline{\tau}_\pi$. Let $\tau^{\mathcal{D}} = \{0, \overline{\tau}_\pi\}$. Per-period payoffs are:

$$v_C(\tau^{\mathcal{D}}, \mathcal{D}) = \frac{\left(1-\alpha\right)AL^2}{2K} - \overline{\tau}_\pi \frac{AL^2}{2}\frac{\left(1-\alpha-K\right)}{K} \tag{1}$$

$$v_W\left(\tau^{\mathcal{D}}, \mathcal{D}\right) = A\left(1-L\right) + \overline{\tau}_\pi \frac{AL^2}{2} \tag{2}$$

$$v_F\left(\tau^{\mathcal{D}}, \mathcal{D}\right) = \frac{\alpha AL^2}{2}\left(1-\overline{\tau}_\pi\right). \tag{3}$$

In a consolidated autocracy, a representative member of the elite solves the following fiscal problem:

$$\max_{\tau_\pi,\tau_L} \frac{\left(1-\alpha\right)AL^2}{2K} - \tau_\pi \frac{AL^2}{2}\frac{\left(1-\alpha-K\right)}{K} + \tau_L LA\left(1-L\right).$$

Clearly, $\tau_L^{\mathcal{A}} = \overline{\tau}_L$. Under the assumption that $1-\alpha > K$, $\tau_\pi^{\mathcal{A}} = 0$. Let $\tau^{\mathcal{A}} = \{\overline{\tau}_L, 0\}$. The per-period payoffs are:

$$v_C(\tau^{\mathcal{A}}, \mathcal{A}) = \frac{\left(1-\alpha\right)AL^2}{2K} + \overline{\tau}_L LA\left(1-L\right) \tag{4}$$

$$v_W(\tau^{\mathcal{A}}, \mathcal{A}) = A\,(1-L)\,(1-\overline{\tau}_L\,(1-L)) \tag{5}$$

$$v_F(\tau^{\mathcal{A}}, \mathcal{A}) = \frac{\alpha A L^2}{2}. \tag{6}$$

1.2 A sufficient condition for democratization to avoid a revolution

If unstable democracy can prevent a revolution, so can fully consolidated democracy and semi-consolidated democracy. Thus, a sufficient condition is that unstable democracy is better for workers than a transition to socialism. Formally, we require

$$\frac{v_W(\mathcal{S})}{1-\beta} - \mu \leq v_W(\tau^{\mathcal{D}}, \mathcal{D}) + \beta W_W(\mathcal{D}). \tag{7}$$

Under the assumption that democracy is unstable, we get

$$W_W(\mathcal{D}) = \psi\left(v_W(\tau^{\mathcal{A}}, \mathcal{A}) + \beta W_W(\mathcal{A})\right) + (1-\psi)\left(v_W(\tau^{\mathcal{D}}, \mathcal{D}) + \beta W_W(\mathcal{D})\right)),$$

where $v_W(\tau^{\mathcal{D}}, D)$ and $v_W(\tau^{\mathcal{A}}, A)$ are given by equations (2) and (5), respectively. Furthermore, we have

$$W_W(\mathcal{A}) = \psi\left(v_W(\tau^{\mathcal{D}}, \mathcal{D}) + \beta W_W(\mathcal{D})\right) + (1-\psi)\left(v_W(\tau^{\mathcal{A}}, \mathcal{A}) + \beta W_W(\mathcal{A})\right).$$

This yields two equations in two unknowns. We can solve to get

$$\begin{aligned} W_W\,(\mathcal{D}) &= \frac{\psi v_W(\tau^{\mathcal{A}}, \mathcal{A}) + (1-\beta(1-2\psi)-\psi) v_W(\tau^{\mathcal{D}}, \mathcal{D})}{(1-\beta\,(1-2\psi))\,(1-\beta)} \\ W_W\,(\mathcal{A}) &= \frac{\psi v_W(\tau^{\mathcal{D}}, \mathcal{D}) + (1-\beta(1-2\psi)-\psi) v_W(\tau^{\mathcal{A}}, \mathcal{A})}{(1-\beta\,(1-2\psi))\,(1-\beta)}. \end{aligned}$$

Substituting this into equation (7) and rearranging yield the condition that

$$\mu > \underline{\mu} = \frac{v_W(\mathcal{S})}{1-\beta} - \frac{v_W(\tau^{\mathcal{D}}, \mathcal{D})}{1-\beta} - \frac{\beta\psi\left(v_W(\tau^{\mathcal{A}}, \mathcal{A}) - v_W(\tau^{\mathcal{D}}, \mathcal{D})\right)}{(1-\beta\,(1-2\psi))\,(1-\beta)}.$$

We assume that this restriction is satisfied.

1.3 The revolution constraint

Suppose the political state is A and assume that $\mu > \underline{\mu}$. Workers never initiate a revolution when the social state is B. If workers initiate a revolution in social state G, there is a transition to socialism and they get

$$V_W(\mathcal{S}) = \frac{v_W(\mathcal{S})}{1-\beta} - \mu.$$

A revolution can be prevented by democratization under the maintained assumption that $\mu > \underline{\mu}$, but tax concessions might be sufficient. Suppose that the elite gives concessions. Clearly, these are only given if $S_t^S = G$. Let $v_W(\tau, \mathcal{A})$ be workers' per-period payoff when the elite offers τ. We have

$$V_W(G, \mathcal{A}) = v_W(\tau, \mathcal{A}) + \beta W_W(\mathcal{A}),$$

where

$$W_W(\mathcal{A}) = \psi\left(v_W(\tau, \mathcal{A}) + \beta W_W(\mathcal{A})\right) + (1-\psi)\left(v_W(\tau^{\mathcal{A}}, \mathcal{A}) + \beta W_W(\mathcal{A})\right).$$

Solving this equation, we get $W_W(A) = \frac{\psi v_W(\tau,\mathcal{A}) + (1-\psi) v_W(\tau^{\mathcal{A}},\mathcal{A})}{1-\beta}$. We can write the revolution constraint as

$$\frac{v_W(\mathcal{S})}{1-\beta} - \mu \leq v_W(\tau, \mathcal{A}) + \beta W_W(\mathcal{A}).$$

Substituting the expression for $W_W(A)$ and rearranging yield

$$v_W(\tau, \mathcal{A}) \geq \frac{\mu(1-\beta) + (1-\psi)\beta v_W(\tau^{\mathcal{A}}, \mathcal{A}) - v_W(\mathcal{S})}{(1-(1-\psi)\beta)}. \quad (8)$$

We can use equation (8) to define two important cut-off values for μ. The first cut-off determines if the elite is forced to democratize to head off a revolution. It is found by letting the elite offer the "maximum" tax concession ($\tau = \tau^{\mathcal{D}}$). This defines

$$\mu_1 = \frac{v_W(\mathcal{S})}{1-\beta} - \frac{(1-(1-\psi)\beta)\, v_W\left(\tau^{\mathcal{D}}, \mathcal{D}\right)}{1-\beta} - \frac{(1-\psi)\beta v_W(\tau^{\mathcal{A}}, \mathcal{A})}{1-\beta} > \underline{\mu}. \quad (9)$$

For $\mu \in (\underline{\mu}, \mu_1)$, tax concessions cannot head off a revolution when $S_t^s = G$ and an extension of the franchise is the only alternative open to the elite. For $\mu \geq \mu_1$, tax concessions can prevent a revolution. Substituting the three expressions for $v_W(.)$ and simplifying yield

$$\mu_1 = \frac{\left(L + 2\beta(1-L)^2(1-\psi)\overline{\tau}_L - L^2(1-\beta(1-\psi))\overline{\tau}_\pi\right)A}{2(1-\beta)} \quad (10)$$

The second cut-off determines if tax concessions are needed or not. It is found by evaluating the revolution constraint at $\tau = \tau^{\mathcal{A}}$ and is given by

$$\begin{aligned} \mu_2 &= \frac{v_W(\mathcal{S})}{1-\beta} - \frac{v_W(\tau^{\mathcal{A}}, \mathcal{A})}{1-\beta} \\ &= \frac{\left(\frac{1}{2}L + \overline{\tau}_L (1-L)^2\right) A}{1-\beta} > \mu_1. \end{aligned} \tag{11}$$

For $\mu > \mu_2$, the revolution constraint is not binding. The elite can implement $\tau^{\mathcal{A}}$ irrespective of the social state at no risk. In the interval $\mu \in [\mu_1, \mu_2]$, tax concessions are necessary and sufficient to avoid a revolution and autocracy can persist. In this case, we say that the autocracy is semi-consolidated.

1.4 The coup constraint

Suppose the political state is D. For the elite, the present value starting in state $(B, \mathcal{D})$ is

$$V_C(B, \mathcal{D}) = v_C(\tau^{\mathcal{D}}, \mathcal{D}) + \beta W_C(\mathcal{D}), \tag{12}$$

where $W_C(D)$ is the continuation value of democracy and $v_C(\tau^{\mathcal{D}}, D)$ is given in equation (1). The continuation value of democracy for the elite is

$$W_C(\mathcal{D}) = \psi V_C(G, \mathcal{D}) + (1-\psi) V_C(B, \mathcal{D}).$$

The present value starting in state $(G, \mathcal{D})$ depends on what the elite does in stage 4. Suppose that it wants to mount a coup if workers propose $\tau^{\mathcal{D}}$ in stage 3. To avoid the coup, workers might give tax concessions, but only when the social state is G. Let $v_C(\tau, \mathcal{D})$ be the one-period payoff of the elite under democracy when the tax vector is τ. We then get

$$V_C(G, \mathcal{D}) = v_C(\tau, \mathcal{D}) + \beta W_C(\mathcal{D}). \tag{13}$$

Given the tax vector τ, the elite does not initiate a coup if

$$v_C(\tau^{\mathcal{A}}, \mathcal{A}) - \phi + \beta W_C(\mathcal{A}) \leq v_C(\tau, \mathcal{D}) + \beta W_C(\mathcal{D}), \tag{14}$$

where $v_C(\tau^{\mathcal{A}}, A)$ is given in equation (4). The continuation value of autocracy, $W_C(A)$, is

$$W_C(\mathcal{A}) = \psi V_C(G, \mathcal{A}) + (1-\psi)\left(v_C(\tau^{\mathcal{A}}, \mathcal{A}) + \beta W_C(\mathcal{A})\right). \tag{15}$$

$V_C(G, \mathcal{A})$ depends on what the elite needs to do to prevent a revolution (they always want to do something, if necessary give away the vote). For $\mu > \underline{\mu}$, democratization clearly dominates a revolution from the point of view of the elite, and it is worse than giving tax concessions. So democratization gives a lower bound on what the elite might get in state $(G, \mathcal{A})$.[1] Suppose the elite democratizes such that

$$V_C\left(G, \mathcal{A}\right) = v_C(\tau^{\mathcal{D}}, \mathcal{D}) + \beta W_C(\mathcal{D}). \tag{16}$$

We can now rewrite the coup constraint by substituting equations (15) and (16) into equation (14) and rearrange:

$$\beta\left(W_C(\mathcal{A}) - W_C(\mathcal{D})\right) \leq \phi + v_C\left(\tau, \mathcal{D}\right) - v_C(\tau^{\mathcal{A}}, \mathcal{A}).$$

Note that

$$W_C(\mathcal{A}) - W_C(\mathcal{D}) = \psi V_C\left(G, \mathcal{A}\right) + (1-\psi) V_C(B, \mathcal{A}) - \left(\psi V_C\left(G, \mathcal{D}\right) + (1-\psi) V_C(B, \mathcal{D})\right). \tag{17}$$

Substitute equations (12), (13) and (16) into (17) and rearrange to get

$$W_C(\mathcal{A}) - W_C(\mathcal{D}) = \frac{(1-\psi)\, v_C(\tau^{\mathcal{A}}, \mathcal{A}) - (1-2\psi)\, v_C(\tau^{\mathcal{D}}, \mathcal{D}) - \psi v_C(\tau, \mathcal{D})}{1-(1-\psi)\,\beta}.$$

Substitute this back into equation (14) to get

$$v_C\left(\tau, \mathcal{D}\right) \geq \frac{v_C(\tau^{\mathcal{A}}, \mathcal{A}) - (1-2\psi)\,\beta v_C(\tau^{\mathcal{D}}, \mathcal{D}) - \left(1-(1-\psi)\,\beta\right)\phi}{1-(1-2\psi)\,\beta}. \tag{18}$$

This is the coup constraint. It defines two important cut-off values of ϕ. The first cut-off determines if workers need to give concessions or not to avoid a coup. Evaluating equation (18) at the tax structure most-preferred by workers, $\tau^{\mathcal{D}} = (0, \overline{\tau}_\pi)$, we get

$$\begin{aligned} \phi_1 &\equiv \frac{v_C(\tau^{\mathcal{A}}, \mathcal{A}) - v_C(\tau^{\mathcal{D}}, \mathcal{D})}{1-(1-\psi)\,\beta} \\ &= \frac{AL\left(2\overline{\tau}_L\,(1-L)^2 + \overline{\tau}_\pi L(L-\alpha)\right)}{2\left(1-(1-\psi)\,\beta\right)(1-L)}. \end{aligned} \tag{19}$$

Given that democratic rights were granted in the past, ϕ_1 is the maximum the elite is willing to pay for a regime change. Accordingly, if $\phi \geq \phi_1$, the coup constraint is irrelevant in the sense that even in the absence of concessions,

[1] Note that democracy cannot arise in the first place if the elite does not democratize to avoid a revolution in social state G.

a coup is not worthwhile. If, on the other hand, $\phi < \phi_1$, workers need to give concessions if they want to avoid a coup.

The second cut-off determines if a coup can be avoided or not. It can be found by evaluating equation (18) at the tax structure most-preferred by the elite, $\tau^{\mathcal{A}} = (\overline{\tau}_L, 0)$:

$$\phi_2 \equiv (1 - 2\psi)\, \beta\phi_1. \tag{20}$$

So, for $\phi \in [0, \phi_2)$, a coup cannot be prevented,[2] but for $\phi \in [\phi_2, \phi_1)$ giving tax concessions in state $S_t^s = G$ is sufficient to make the elite indifferent between a coup and continued democracy.

1.5 Propositions

We shall state the equilibrium regime configurations and the associated tax structures in two propositions. The first proposition characterizes different types of autocracies while the second characterizes different types of democracies. As a pre-ample to the proofs of the two propositions, we must define the strategies of the two players (the workers and the elite) and the equilibrium concept. The state of the economy is either $\left(S^S, \mathcal{A}\right)$, (S^S, D) or S where $S^S \in \{G, B\}$. When the state is (S^S, D), a strategy of the elite is a function of the state and workers' choice of tax structure. When the state is (S^S, A), it is a function only of the state. The strategy determines the optimal action of the elite in each state. In state (S^S, D), the action space of the elite is to mount a coup or not, and if a coup is mounted, to decide a tax structure. In state (S^S, A), the action space of the elite consists of a decision to democratize or not, and in the absence of democratization, a decision on the tax structure. Since state S is absorbing, we need not specify the strategy of the elite in this state. When the state is (S^S, A), a strategy of workers is a function of the state of the world, the elite's decision to introduce democracy or not, and of the elite's proposed tax structure. When the state is (S^S, D), workers' strategy is simply a function of the state. The strategy determines the appropriate action of workers. In state (S^S, A), their action space is a decision to mount a revolution or not, while in state (S^S, D), workers need to decide on the tax structure only. A pure strategy Markov perfect equilibrium is defined as a set of strategies for workers and the elite that are best responses to each other for all possible states.

1.5.1 Autocracies

The first proposition characterizes the three different types of fully consolidated or semi-consolidated autocracy that can arise along a Markov perfect

[2]The assumption that $\psi < \frac{1}{2}$ implies that $\phi_2 > 0$.

equilibrium path.

Proposition 1 *(Autocracy) Suppose that the destination country is initially an autocracy and that $\mu \geq u_1$. The domestic economy remains an autocracy. The tax structure is $\tau^{\mathcal{A}}$ in social state B. Moreover, there exists a $\mu_3 \in (\mu_1, \mu_2)$ such that*

1. If $\mu \in [\mu_1, \mu_3)$, then the autocracy is semi-consolidated. The tax structure is FDI-unfriendly and progressive ($\tau_L > 0$, $\tau_\pi = \overline{\tau}_\pi$) in social state G.

2. If $\mu \in [\mu_3, \mu_2]$, then the autocracy is semi-consolidated. The tax structure is FDI-friendly and regressive ($\tau_L = \overline{\tau}_L$, $\tau_\pi < \overline{\tau}_\pi$) in social state G.

3. If $\mu > \mu_2$, then the autocracy is fully consolidated. The tax structure is always $\tau^{\mathcal{A}}$, i.e., FDI-friendly and regressive.

We can prove proposition 1 as follows. Assume that $\mu \geq \mu_1$. The initial political state is autocracy, i.e., $S^P = A$. We can solve for the complete set of Markov perfect equilibria by backward induction. In autocracy, workers move after the elite. In state $(B, \mathcal{A})$, the unique best response of workers to any τ set by the elite is not to stage a revolution. The optimal response of the elite is to set $\tau = \tau^{\mathcal{A}}$ and not to democratize. In state $(G, \mathcal{A})$, workers' best response to a tax structure that makes them at least as well off as they would be by undertaking a revolution is not to stage a revolution. Their best response to a tax structure that fails to achieve this is to stage a revolution. Moreover, the best response to democratization is not to stage a revolution. Anticipating this, the elite's unique best response is to prevent a revolution by giving the minimum concession required to avoid a revolution. How this is done is described by the following lemma.

Lemma 1 *Suppose that the state is (G, A). There exists a value $\mu_3 \in (\mu_1, \mu_2)$ such that it is optimal for the elite to offer the following tax concessions in state G:*

1. *For $\mu > \mu_2$, the revolution constraint is not binding. The elite sets $\tau^{\mathcal{A}} = (\overline{\tau}_L, 0)$.*

2. *For $\mu \in [\mu_3, \mu_2]$, the revolution constraint is binding. The elite sets $\tau_L = \overline{\tau}_L$ and $\tau_\pi(\mu) \in [0, \overline{\tau}_\pi]$ with $\tau_\pi(\mu_2) = 0$, $\tau_\pi(\mu_3) = \overline{\tau}_\pi$ and $\tau'_\pi(\mu) < 0$.*

3. *For $\mu \in [\mu_1, \mu_3)$, the revolution constraint is binding. The elite sets $\tau_L(\mu) = [0, \overline{\tau}_L)$ and $\tau_\pi = \overline{\tau}_\pi$ with $\tau_L(\mu_1) = 0$ and $\tau_L' > 0$.*

Proof *Clearly tax rates will never exceed $\overline{\tau}_L$ and $\overline{\tau}_\pi$. Given that the problem solved by the elite is*

$$\max_{\tau_L, \tau_\pi} \frac{(1-\alpha) AL^2}{2K} - \tau_\pi \frac{AL^2}{2} \frac{(1-\alpha-K)}{K} + \tau_L LA (1-L)$$

subject to the revolution constraint (equation (8)). We can write the revolution constraint as

$$A(1-L) - \tau_L A (1-L)^2 + \tau_\pi \frac{AL^2}{2} - Q(\mu) \geq 0,$$

where

$$Q(\mu) = -\frac{\mu(1-\beta) + (1-\psi)\beta v_W(\tau^{\mathcal{A}}, \mathcal{A}) - v_W(\mathcal{S})}{(1-(1-\psi)\beta)}.$$

The first order derivatives are:

$$\begin{aligned} \tau_L &: A(1-L)(1-\xi_r K) \\ \tau_\pi &: \frac{AL^2}{2}\left(\xi_r - \frac{1-\alpha-K}{K}\right) \end{aligned}$$

where ξ_r is the Lagrangian multiplier on the revolution constraint. For $\mu > \mu_2$, we have that $Q(\mu) < v_W(\tau^{\mathcal{A}}, A)$ and the unconstrained optimal tax structure $\tau^{\mathcal{A}} = \{\overline{\tau}_L, 0\}$ is sufficient to avoid a revolution. Moreover, for $\mu < \mu_1$, not even $\tau^{\mathcal{D}} = \{0, \overline{\tau}_\pi\}$ can prevent a revolution. Define μ_3 as the solution to $v_W(\tau^{"}, A) = Q(\mu)$. Solving this gives

$$\begin{aligned} \mu_3 &= \frac{(1-\psi)\beta\left(v_W(\tau^{"}, \mathcal{A}) - v_W(\tau^{\mathcal{A}}, \mathcal{A})\right)}{1-\beta} - \frac{v_W(\tau^{"}, \mathcal{A}) - v_W(\mathcal{S})}{1-\beta} \qquad (21) \\ &= \frac{\left(L - L^2\overline{\tau}_\pi + 2(1-L)^2(1-\beta(1-\psi))\overline{\tau}_L\right)A}{2(1-\beta)} \end{aligned}$$

where $\tau^{"} = \{\overline{\tau}_L, \overline{\tau}_\pi\}$. Notice that the marginal cost to the elite of providing on unit of income to workers to meet the revolution constraint through a cut in the wage tax, $\frac{1}{K}$, is larger than the marginal cost of doing it through an increase in the tax on profit, $\frac{1-\alpha-K}{K}$. This implies that for $\mu \in [\mu_3, \mu_2]$, the elite chooses to satisfy the revolution constraint by leaving $\tau_L = \overline{\tau}_L$ and setting

$$\tau_\pi(\mu) = 2\left(\overline{\tau}_L A(1-L)^2 - A(1-L) + Q(\mu)\right)/AL^2$$

with $\tau_\pi'(\mu) < 0$ and $\tau_\pi(\mu_3) = \overline{\tau}_\pi$. For $\mu \in [\mu_1, \mu_3)$, the elite must set $\tau_\pi = \overline{\tau}_\pi$ and, in addition, cut the wage tax to meet the constraint. The wage tax is

$$\tau_L(\mu) = \frac{1}{A\,(1-L)^2}\left\{A\,(1-L) + \tau_\pi \frac{AL^2}{2} - Q(\mu)\right\}$$

with $\tau_\pi'(\mu) > 0$ and $\tau_L(\mu_1) = 0$ ∎

The three types of autocracy and the associated tax structures follow immediately from this.

1.5.2 Democracies

The next proposition characterizes the four different types of democracy that can emerge along a Markov perfect equilibrium path.

Proposition 2 *(Democracy) Suppose that the destination country is initially an autocracy and that $\underline{\mu} < \mu < \mu_1$. There exists a $\phi_3 \in (\phi_2, \phi_1)$ such that the following is true:*

1. If $\phi < \phi_2$, the destination country becomes an unstable democracy that switches regime each time the social state is G. The tax structure oscillates between periods with $\tau^{\mathcal{D}} = (0, \overline{\tau}_\pi)$ and periods with $\tau^{\mathcal{A}} = (\overline{\tau}_L, 0)$.

2. If $\phi \in [\phi_2, \phi_3]$, the destination country becomes a semi-consolidated democracy the first time the state is $(G, \mathcal{A})$. In social state B, the tax structure is $\tau = \tau^{\mathcal{D}}$. In social state G, the tax structure is FDI-friendly: $\tau_L = \overline{\tau}_L$ and $\tau_\pi(\phi) \leq \overline{\tau}_\pi$ with $\tau_\pi'(\phi) > 0$, $\tau_\pi(\phi_2) = 0$ and $\tau_\pi(\phi_3) = \overline{\tau}_\pi$.

3. If $\phi \in (\phi_3, \phi_1)$, the destination country becomes a semi-consolidated democracy the first time the state is $(G, \mathcal{A})$. In social state B, the tax structure is $\tau = \tau^{\mathcal{D}}$. In social state G, the tax structure is FDI-unfriendly: $\tau_\pi = \overline{\tau}_\pi$ and $\tau_L(\phi) > 0$ with $\tau_L'(\phi) < 0$.

4. If $\phi \geq \phi_1$, the destination country becomes a consolidated democracy the first time the state is $(G, \mathcal{A})$. The resulting tax structure is always FDI-unfriendly: $\tau_L = 0$ and $\tau_\pi = \overline{\tau}_\pi$.

We can solve for the complete set of Markov perfect equilibria by backward induction. In autocracy, workers move after the elite. In state $(B, \mathcal{A})$, the unique best response of workers to any τ set by the elite is not to stage a revolution. Given that the optimal response of the elite is to set $\tau = \tau^{\mathcal{A}}$ and not to democratize. In state $(G, \mathcal{A})$, a revolution cannot be prevented by any tax concessions. The best response of workers is to stage a revolution if the elite does not democratize and not to stage one if it does introduce democracy. The elite's unique best response to this is to democratize. This leads to a transition to state $(G, \mathcal{D})$ and workers set $\tau = \tau^{\mathcal{D}}$ immediately after the transition (there is not threat of a coup). Now consider state $(B, \mathcal{D})$. In democracy, workers move before the elite. When the social state is B, the unique best response of the elite to any tax structure chosen by workers is not to mount a coup. Anticipating this, the unique best response of workers is to set $\tau = \tau^{\mathcal{D}}$. Consider state $(G, \mathcal{D})$. First, suppose that $\phi \geq \phi_1$. By construction, the coup constraint does not bind. This means that the best response to any tax structure proposed by workers is not to mount a coup. Workers' unique best response is to set $\tau = \tau^{\mathcal{D}}$. The result is fully consolidated democracy. Second, suppose that $\phi \in [\phi_2, \phi_1)$. Given the tax structure τ', the best response of the elite is to mount a coup if

$$v_C(\tau^{\mathcal{A}}, \mathcal{A}) - \phi + \beta W_C(\mathcal{A}) > v_C(\tau', \mathcal{D}) + \beta W_C(\mathcal{D}). \tag{22}$$

The unique best response is not to mount a coup if this condition fails. To derive the best response to this from workers, we begin by characterizing the least costly tax concession required to make the coup constraint bind.

Lemma 2 *Suppose that the political state is (G,D). There exists a value $\phi_3 \in (\phi_2, \phi_1)$ such that the least costly tax concession that avoids a coup is:*

1. *For $\phi \in (\phi_3, \phi_1)$, workers set $\tau_\pi = \overline{\tau}_\pi$ and $\tau_L(\phi) > 0$ with $\tau_L'(\phi) < 0$.*

2. *For $\phi \in [\phi_2, \phi_3]$, workers set $\tau_L = \overline{\tau}_L$ and $\tau_\pi(\phi) \leq \overline{\tau}_\pi$ with $\tau_\pi'(\phi) > 0$, $\tau_\pi(\phi_2) = 0$ and $\tau_\pi(\phi_3) = \overline{\tau}_\pi$.*

Proof *Clearly tax rates will never exceed $\overline{\tau}_L$ and $\overline{\tau}_\pi$. Given that, the fiscal problem that the workers face is:*

$$\max_{\tau_L, \tau_\pi} A(1-L) - \tau_L A(1-L)^2 + \tau_\pi \frac{AL^2}{2}$$

subject to the coup constraint given by

$$\frac{(1-\alpha) AL^2}{2K} - \tau_\pi \frac{AL^2}{2} \frac{(1-\alpha-K)}{K} + \tau_L LA(1-L) - O(\phi) \geq 0,$$

where

$$O(\phi) = \frac{v_C(\tau^{\mathcal{A}}, \mathcal{A}) - (1-2\psi)\beta v_C(\tau^{\mathcal{D}}, \mathcal{D}) - (1-(1-\psi)\beta)\phi}{1-(1-2\psi)\beta}.$$

The first derivatives are

$$\begin{aligned}\tau_L &: AK(\xi_c - K) \\ \tau_\pi &: \frac{AL^2}{2}(1-\xi_c\frac{(1-\alpha-K)}{K})\end{aligned}$$

where ξ_c *is the Lagrangian multiplier associated with the coup constraint. Let* ϕ_3 *be defined as the solution to* $v_C(\tau', D) = O(\phi)$*, i.e.,*

$$\begin{aligned}\phi_3 &= \frac{\beta(1-2\psi)(v_C(\tau^{'},\mathcal{D}) - v_C(\tau^{\mathcal{D}},\mathcal{D}))}{1-(1-\psi)\beta} + \frac{v_C(\tau^{\mathcal{A}},\mathcal{A}) - v_C(\tau^{'},\mathcal{D})}{1-(1-\psi)\beta} \quad (23) \\ &= \phi_1 - \frac{(1-L)(1-\beta(1-2\psi))AL\overline{\tau}_L}{(1-(1-\psi)\beta)}\end{aligned}$$

where $\tau^{'} = (\overline{\tau}_L, \overline{\tau}_\pi)$*. We note that* $\phi_1 > \phi_3 > \phi_2$*. From the two first derivatives, we notice that the cost to workers of providing one unit of income to the elite to meet the coup constraint through an increase in the wage tax,* K*, is smaller than the marginal cost of doing it through a decrease in the tax on profit,* $\frac{K}{1-\alpha-K}$*. For* $\phi \in (\phi_3, \phi_1)$*, it therefore follows that workers set* $\tau_\pi = \overline{\tau}_\pi$ *and increase the tax on wages to*

$$\tau_L(\phi) = \frac{1}{LA(1-L)}\left(\overline{\tau}_\pi\frac{AL^2}{2}\frac{(1-\alpha-K)}{K} + O(\phi) - \frac{(1-\alpha)AL^2}{2K}\right)$$

with $\tau_L'(\phi) < 0$*. For* $\phi \in [\phi_2, \phi_3]$*, workers set* $\tau_L = \overline{\tau}_L$ *and*

$$\tau_\pi(\phi) = \frac{2K}{AL^2(1-\alpha-K)}\left(\frac{(1-\alpha)AL^2}{2K} + \overline{\tau}_L LA(1-L) - O(\phi)\right)$$

with $\tau_\pi'(\phi) > 0$*,* $\tau_\pi(\phi_2) = 0$ *and* $\tau_\pi(\phi_3) = \overline{\tau}_\pi$ ■

The next lemma establishes the conditions under which giving the least costly tax concession constitutes a best response to the elite's strategy of mounting a coup if condition (22) fails.

Lemma 3 *Suppose that* $\psi < \frac{1}{2}$*. Then giving the least costly tax concession to avoid a coup is a best response for workers for all* $\phi \geq \phi_2$*.*

Proof *A sufficient condition is that workers are willing to give the maximum concession* $\tau^{\mathcal{A}}$ *in state* G *rather than accepting an unstable democracy in which there is a coup every time the state is* $(G, \mathcal{D})$ *and enfranchisement every time the state is* $(G, \mathcal{A})$*. Let the expected present value for a worker in state* $(G, \mathcal{D})$ *when the maximum concession is given be denoted* $V_W(G, \mathcal{D}|\,\tau^{\mathcal{A}})$*. We get that*

$$V_W(G,\mathcal{D}|\,\tau^{\mathcal{A}}) = v_W(\tau^{\mathcal{A}}) + \beta\psi V_W(G,\mathcal{D}|\,\tau^{\mathcal{A}}) + \beta(1-\psi)V_W(B,\mathcal{D}), \quad (24)$$

where

$$V_W(B,\mathcal{D}) = \frac{v_W\left(\tau^{\mathcal{D}},\mathcal{D}\right) + \beta\psi V_W(G,\mathcal{D}|\,\tau^{\mathcal{A}})}{1-\beta(1-\psi)}$$

Substituting this into equation (24), we get

$$V_W(G,\mathcal{D}|\,\tau^{\mathcal{A}}) = \frac{(1-\beta(1-\psi))\,v_W\left(\tau^{\mathcal{A}},\mathcal{A}\right) + \beta(1-\psi)\,v_W\left(\tau^{\mathcal{D}},\mathcal{D}\right)}{1-\beta}.$$

The expected present value for a worker in state $(G,\mathcal{D})$ *when a coup leads to unstable democracy (the most benign type of autocracy) is*

$$V_W(G,\mathcal{D}) = v_W\left(\tau^{\mathcal{A}},\mathcal{A}\right) + \beta\psi V_W(G,\mathcal{A}) + \beta(1-\psi)\,V_W(B,\mathcal{A}), \qquad (25)$$

where

$$V_W(G,\mathcal{A}) = v_W\left(\tau^{\mathcal{D}},\mathcal{D}\right) + \beta\psi V_W(G,\mathcal{D}) + \beta(1-\psi)\,V_W(B,\mathcal{D})$$

and

$$V_W(B,\mathcal{A}) = \frac{v_W\left(\tau^{\mathcal{A}},\mathcal{A}\right) + \beta\psi V_W(G,\mathcal{A})}{1-\beta(1-\psi)}.$$

Note that $V_W(B,\mathcal{D}) = V_W(G,\mathcal{A})$. *This implies that*

$$V_W(G,\mathcal{A}) = \frac{v_W\left(\tau^{\mathcal{D}},\mathcal{D}\right) + \beta\psi V_W(G,\mathcal{D})}{1-\beta(1-\psi)}.$$

Substitution in equation (25) gives

$$\begin{aligned} V_W(G,\mathcal{D}) &= \frac{(1-\beta(1-\psi))}{(1-\beta)(1-\beta(1-2\psi))} v_W\left(\tau^{\mathcal{A}},\mathcal{A}\right) \\ &\quad + \frac{\beta\psi}{(1-\beta)(1-\beta(1-2\psi))} v_W\left(\tau^{\mathcal{D}},\mathcal{D}\right) \end{aligned}$$

We seek a condition such that

$$V_W(G,\mathcal{D}|\,\tau^{\mathcal{A}}) > V_W(G,\mathcal{D})$$

The difference $V_W(G,\mathcal{D}|\,\tau^{\mathcal{A}}) - V_W(G,D)$ *can be written as*

$$\left(\frac{\beta(1-2\psi)(1-\beta(1-\psi))}{(1-\beta(1-2\psi))}\right)\left(\frac{v_W\left(\tau^{\mathcal{D}},\mathcal{D}\right)}{1-\beta} - \frac{v_W\left(\tau^{\mathcal{A}},\mathcal{A}\right)}{1-\beta}\right).$$

Since $v_W\left(\tau^{\mathcal{D}},\mathcal{D}\right) > v_W\left(\tau^{\mathcal{A}},\mathcal{A}\right)$ *this expression is positive if and only if* $\frac{(1-2\psi)}{(1-\beta(1-2\psi))} > 0 \Leftrightarrow \psi < \frac{1}{2}$ ■

Since our maintained assumption is that $\psi < \frac{1}{2}$, we conclude from the lemma that it is a best response for workers to give concessions when this is necessary and sufficient to avoid a coup, i.e., for $\phi \in [\phi_2, \phi_1)$. This gives rise to the two different types of semi-consolidated democracy listed in the proposition.

Thirdly, suppose that $\phi < \phi_2$. Notice that $\phi < \phi_2 \Leftrightarrow O(\phi) > v_C(\tau^{\mathcal{A}}, A)$. This implies that no matter what tax structure workers propose, the unique best response of the elite is to stage a coup. The society moves to state $(G, \mathcal{A})$ where the elite sets $\tau = \tau^{\mathcal{A}}$ (no threat of revolution immediately after a coup). Following that players follow the optimal strategies for state $(\cdot, \mathcal{A})$ specified above. This implies that the elite democratizes each time the state is $(G, \mathcal{A})$. The society becomes an unstable democracy.

Online appendix B

This appendix derives the comparative statics with respect to γ_ϕ, γ_μ and $\phi(\mu)$. The definitions of γ_ϕ, γ_μ and $\phi(\mu)$ are given in equations (14), (15) and (16) in the printed text. Substitution of ϕ_2 and μ_2 into equation (16) yields:

$$\begin{aligned}\phi(\mu) &= \frac{\left(\overline{\tau}_L(1-L)+\overline{\tau}_\pi\frac{L}{2}\frac{(L-\alpha)}{1-L}\right)(1-2\psi)\beta AL}{1-(1-\psi)\beta}-\frac{AL^2\alpha\overline{\tau}_\pi\gamma}{2\beta\psi}\\ &\quad -\frac{(1-\beta(1-2\psi)\left(\mu-A\frac{\frac{1}{2}L+\overline{\tau}_L(1-L)^2}{1-\beta}\right)}{\beta\psi}.\end{aligned}$$

Taking the derivative with respect to A gives:

$$\frac{\partial\gamma_\mu}{\partial A}=\frac{\partial\gamma_\phi}{\partial A}=0.$$

Evaluate $\phi(\mu)$ at μ_1 and calculate

$$\frac{\partial\phi(\mu_1)}{\partial A}=\frac{\left(\overline{\tau}_L(1-L)+\overline{\tau}_\pi\frac{L}{2}\frac{(L-\alpha)}{1-L}\right)(1-2\psi)\beta L}{1-(1-\psi)\beta}-\frac{L^2\alpha\overline{\tau}_\pi\gamma}{2\beta\psi}.$$

This derivative is negative for γ sufficiently large. It may, however, be positive. This is, for example, the case for $\gamma=\gamma_\phi>\gamma_\mu$ at $\beta=\frac{9}{10},\overline{\tau}_L=\frac{9}{10},\overline{\tau}_\pi=\frac{9}{10},L=\frac{3}{4},\alpha=\frac{1}{2},\psi=\frac{1}{10}$.

Taking the derivative with respect to α gives:

$$\begin{aligned}\frac{\partial\gamma_\mu}{\partial\alpha} &= \frac{-\left(\overline{\tau}_L 2(L-1)^2+L^2\overline{\tau}_\pi\right)(1-\beta+\beta\psi)^2}{(1-\beta)L^2\alpha^2\overline{\tau}_\pi}<0\\ \frac{\partial\gamma_\phi}{\partial\alpha} &= \frac{-\left(\overline{\tau}_L 2(L-1)^2+L^2\overline{\tau}_\pi\right)(1-\beta+2\beta\psi)}{(1-L)L\alpha^2\overline{\tau}_\pi}<0.\\ \frac{\partial\phi(\mu)}{\partial\alpha} &= -(1-2\psi)\beta AL\frac{\overline{\tau}_\pi\frac{1}{2}\frac{L}{1-L}}{1-(1-\psi)\beta}-\frac{AL^2\overline{\tau}_\pi\gamma}{2\beta\psi}<0.\end{aligned}$$

Taking the derivative with respect to L gives:

$$\begin{aligned}\frac{\partial\gamma_\mu}{\partial L} &= \frac{-4(1-L)(1-\beta(1-\psi))^2\overline{\tau}_L}{(1-\beta)L^3\alpha\overline{\tau}_\pi}<0\\ \frac{\partial\gamma_\phi}{\partial L} &= -\frac{(1-\beta+2\beta\psi)\left(2\overline{\tau}_L(1-L)^2-\overline{\tau}_\pi L^2(1-\alpha)\right)}{(1-L)^2L^2\alpha\overline{\tau}_\pi}\end{aligned}$$

which is negative **if** $2>\frac{\overline{\tau}_\pi}{\overline{\tau}_L}\frac{L^2(1-\alpha)}{(1-L)^2}$.