25

Growth and Capital

25.1 THE THEORY OF GROWTH

Harrod and Domar

Modern growth theory stems from the work of Harrod (1939) and Domar (1946). Though the equation central to both their models has come to be called the Harrod–Domar model,[1] they approached the problem of growth very differently.

Harrod's theory centred on the concept of the *warranted rate of growth*:

> that rate of growth which, if it occurs, will leave all parties satisfied that they have produced neither more nor less than the right amount ... it will put them into a frame of mind which will cause them to give such orders as will maintain the same rate of growth.[2]

The relationship of supply and demand was analysed, in a Keynesian manner, in terms of savings and investment. Saving was assumed to be a fraction, s, of output. Investment was determined by the acceleration principle, and depended on the growth of output. Defining v as "the value of capital goods required for the production of a unit increment of output",[3] equality of planned savings and investment requires that

$$sY = vY,$$

which implies that the growth rate, Y/Y, equals s/v. This growth rate was Harrod's warranted rate of growth.

Harrod went on to draw pessimistic conclusions about the possibility of steady growth. The main problem was that divergences of the actual growth rate from the warranted rate would be cumulative. If, for example, the actual growth rate were less than the warranted rate, the result would be that producers would find themselves with too much capital (either unwanted inventories, or too much equipment) and they would reduce their investment. The result of this would be that the growth rate would, because of the multiplier effect on income, fall even further below the warranted rate. This problem of the instability of the warranted rate was reinforced by the existence of a limit imposed on the actual rate of growth by what Harrod called the *natural rate of growth*. This was

> the maximum rate of growth allowed by the increase of population, accumulation of capital, technological improvement and the work/leisure preference schedule, supposing that there is always full employment in some sense.[4]

If the natural rate were less than the warranted rate (and there was no reason why this should not be the case) then depression would result, for the economy could not possibly expand at the warranted rate for very long.

Although s and v are treated as constants in Harrod's algebra, he does not assume them to be fixed. Harrod refers to investment depending on the rate of interest, and on long term plans as well as on the increase in output.[5] The level of savings will depend not only on the level of output, but also on the level of unemployment. Thus the warranted rate of growth (s/v) might be "dragged down by depression", or be "twisted upwards by an inflation of prices and profit".[6]

Domar's starting point, on the other hand, was the dual role of investment.[7] On the one hand it produces demand, via the multiplier, and on the other it increases productive capacity. Domar was thus concerned to find the conditions under which demand would grow at the same rate as productive capacity. He argued that it was important to see unemployment as dependent not on the level of national income, but on the ratio of income to productive capacity.[8] He distinguished between unemployment caused by deficient demand, and that due to the productivity of investment being less than its maximum value. The latter might arise, for example, if investment were inefficiently allocated between different uses. Domar came to the conclusion that the economy would converge towards a degree of capacity utilization given by the ratio of the growth rate of investment to the required growth rate, the latter being given by the same formula as Harrod's warranted growth rate.[9] A low rate of growth would thus lead to permanent stagnation.

It is important to note that although Harrod and Domar both derived the equation that has become associated with their names, they used it in very different ways. Firstly, their interpretations of v were different. Domar postulated a direct link between v and increases in productive capacity, something which Harrod did not do: for Harrod the accelerator was a theory of the demand for investment.[10] Secondly, they had completely different accounts of what went on when the economy was not growing at the warranted, or required, growth rate.

Neoclassical growth theory

Although Harrod and Domar recognized that s and v would in practice vary, their formal models treated them as constants. In the 1950s economists tried to construct formal models which allowed for changes in s and v. The predominant approach was the "neoclassical" one, this originating in papers by Solow (1956) and Swan (1956). Solow, for example, criticized Harrod and Domar for studying the problem of the long run with short run tools. Harrod and Domar had, according to Solow, assumed fixed coefficients whereas, argued Solow, "One usually thinks of the long run as the domain of the neoclassical analysis, the land of the margin."[11] Solow proposed to accept all the Harrod–Domar assumptions, except that of the

constant capital–output ratio.[12] There is, however, much more to Solow's model than this, for he assumes perfect competition, an assumption not made by either Harrod or Domar. In Solow's model the economy is a miniature general equilibrium model, with competitive equilibrium prevailing in the markets for labour, capital and output. At any moment there are given stocks of capital and labour, with competition ensuring full employment, and the equality of factor prices to marginal products. Output is linked to inputs of capital and labour by a production function exhibiting with marginal products. Growth comes about because a fraction of output is saved and invested, this increasing the capital stock. The labour force is assumed to be growing at an exogenously given rate. If capital accumulates at a rate different from the rate at which the labour force is growing, the capital–labour ratio will change, this in turn causing factor prices to change. Solow was able to show that, starting from any arbitrary capital stock, the economy would converge on an equilibrium where capital and labour were growing at the same rate, and where factor prices were constant.

When Solow's model, which was essentially the same as Swan's, is compared with the earlier models of Harrod and Domar, stress is often laid on the production function which allowed capital to be substituted for labour, but this is not the main difference. The significant point about Solow's model is that it tackled the problem of growth from the point of view of general equilibrium theory: as mentioned above, it is a miniature competitive equilibrium model, with competition ensuring that supply and demand are equal in all markets, all the time. It is a dynamic version of the model analysed by Hicks in *The Theory of Wages* (1932), a model in which competition ensures that Keynesian problems do not arise. This is in marked contrast to the models of both Harrod and Domar which, for all their differences, both claimed to say something about the course of unemployment over time.

Much of the vast literature appearing on growth theory in the 1960s was concerned with relaxing the heroic abstractions made in Solow's one-sector model. The most obvious was to disaggregate, the first step being to construct models with two sectors, one producing consumption goods, the other investment goods (Meade, 1961; Uzawa, 1961). This model worked in much the same way as Solow's model, though making the general equilibrium aspects of the model more explicit: supply and demand determine the relative prices of the two goods as well as factor prices.[13] Attempts were made to consider alternative technologies, and to allow for technical progress in a more realistic way – for example, so called "vintage models", in which the productivity of capital goods is assumed to depend on when they were made, newer ones being more productive than old ones. However, whilst there was no problem in extending the model to cover a wide variety of consumption goods, problems emerged when a variety of capital goods was introduced.

One of the most significant problems is what has become known as the "Hahn problem" (Hahn, 1966). The essence of this problem is that the value of a capital good depends on its yield, this including any capital gains from

holding the capital good. A rise in the price of any capital good will result in a capital gain, this raising the yield and hence raising the demand still further, giving a further capital gain. The instability this causes was found to apply to a wide class of models.

An alternative approach was to move closer to macroeconomics, introducing money explicitly into growth models. The most common approach here, though there were exceptions, was to postulate a government which financed its debts by issuing debt which the private sector purchased. Instead of holding only capital, the private sector now held two assets, capital and government debt, demands for these two assets depending on their relative yields. Though its only function was to be held as an asset, this government debt was called "money". Some of the economists who analysed this type of model retained the competitive equilibrium assumptions (e.g. Tobin, 1965; Sidrauski, 1969), whereas others departed from the competitive equilibrium framework by allowing markets to be out of equilibrium, admitting the possibility of unemployment. (e.g. Rose, 1973; Uzawa, 1973; Stein, 1971). Within this framework an extra variable, the growth rate of the money supply, was introduced, and it was possible to analyse the effects of this on the equilibrium growth path.

In virtually all this literature, the long run equilibrium growth rate was taken as exogenous, determined by the rate of population growth and the rate of growth of productivity. What then did growth models show? (1) They showed how an economy might, starting from an arbitrary starting point, move towards an equilibrium growth path on which the growth rate would be that set by population growth and technical progress. (2) Although the equilibrium *growth rate* was exogenous, other characteristics of the long run equilibrium, such as consumption per head, did depend on parameters such as the propensity to save, or, in "monetary" models, the growth rate of the money supply. These relationships could be investigated.

Von Neumann, Ramsey and optimal growth

An alternative approach to the problem of production, which avoids the problem of measuring capital, is to deal with capital as a list of specific physical capital goods. The seminal paper dealing with growth within this framework was that of von Neumann (1938). This paper used a linear model of production. There was a set of productive processes, each using a specified list of inputs to produce a specified list of outputs. No capital aggregate was needed. The problem of fixed capital was dealt with by including old capital goods in the list of outputs. This means that, for example, a process might comprise:

$$1 \text{ unit labour} + 1 \text{ new machine} \rightarrow 1 \text{ unit food} + 1 \text{ old machine.}$$

To obtain a growth model von Neumann assumed that the supply of labour was perfectly elastic at a fixed, subsistence wage rate, and that all profits were invested. His model, apart from the absence from it of any scarce resources, thus had a very classical flavour.

Von Neumann went on to prove that there existed a balanced growth path along which the economy could expand, and to find the growth path with the highest growth rate. He was able to show that on this growth path with the maximal growth rate: (1) there would be a set of prices and a rate of interest such that, if they prevailed, a competitive economy would grow at this growth rate; and (2) this interest rate would equal the growth rate.[14]

A natural development from this model was that economists tried to generalize it, and to analyse similar models, such as Leontief's, one based on a simpler technology than von Neumann's model. For example, if the assumption of a subsistence wage rate were dropped, it was possible to show that as the real wage rose, the von Neumann growth rate fell. Thus if the growth rate of the labour force were fixed, it was possible to find the real wage rate at which the economy (and hence demand for labour) would grow at the same rate as the supply of labour. If the assumption that all profits were saved was relaxed, it followed that the rate of growth was equal to the rate of interest multiplied by the fraction of profits saved.

The von Neumann model raised the question of optimal growth, which is where we need to bring in another pre-war contribution, that of Ramsey (1927). Like von Neumann, Ramsey was concerned with optimal growth, but where von Neumann was concerned to find the highest growth rate consistent with the technology, Ramsey tackled the problem from the consumers' point of view. Ramsey's problem involved finding the path of capital accumulation, not necessarily with a constant growth rate, which maximized the present value of consumers' utility. Much work was done in the 1950s and 1960s on the problem of optimal growth, both capital orientated (von Neumann's problem of maximizing the rate of capital accumulation) and consumption orientated (Ramsey's problem), such problems being seen as relevant to the problem of planning. Several contributions to this literature are worth picking out. An influential early paper was Malinvaud (1953), which looked at the relationship of Pareto efficiency and competitive equilibrium in a model with an infinite time horizon.[15] Another important contribution was that of Dorfman, Samuelson and Solow (1958), which brought the notion of the "turnpike" to the notice of less mathematical economists.[16] Though proofs of turnpike theorems usually involve complicated mathematics, the basic idea is very simple. Suppose that there are two goods, x and y, and that an economy starts with initial stocks of x_0 and y_0 in Figure 25.1. Assume the objective is to accumulate as much capital as possible by time T, but to retain the initial ratio of x to y. In other words the objective is to move as far as possible along the line OR. The problem is what path to follow. Now assume the von Neumann path is ON. This is the balanced growth path along which the growth rate is maximized. The turnpike theorem says that the optimal path will be one which goes towards the von Neumann path (the "turnpike", of "motorway"), moves along it for a while, and then returns towards the target shortly before time T. In the same way that it pays a driver to use a turnpike even if it is not the most direct route, if her journey is sufficiently long, so it is optimal for the economy to grow along the von Neumann growth path if the time horizon is sufficiently long. It could thus

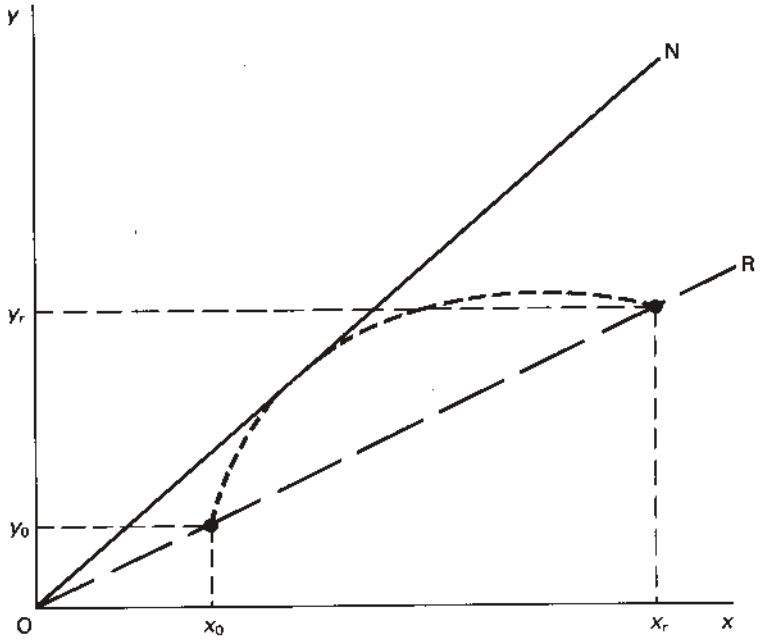

FIGURE 25.1 *The Turnpike*

be argued that the turnpike theorem gives relevance to the von Neumann growth path, for there is, in general, no reason to assume either that planners will desire balanced growth for its own sake, or that the economy will have exactly the right initial stocks to follow the von Neumann path immediately.

When approaching the problem of optimal growth from the consumption side, contact was made with aggregative growth models. The main result here was that derived simultaneously by Robinson (1962b), who called it the "neoclassical theorem", and Phelps (1961), whose name "the golden rule of accumulation" became the accepted one. The golden rule deals with the problem of maximizing steady state consumption per head, stating that it will be maximized if the rate of interest, equal to the marginal product of capital, equals the growth rate.[17] Planners may not wish to maximize steady state consumption per head, but it was shown that the golden rule growth path was related to Ramsey's more general problem in much the same way that the von Neumann growth path was to the capital orientated planning problem discussed above.

"Keynesian" growth models

Most of the growth models discussed so far fall under the heading "neoclassical".[18] Throughout the period, however, a small but widely

noticed group of economists argued against this approach, with its assumptions of smooth factor substitution and competitive markets, putting forward alternative theories of growth and distribution. The main members of this group were Robinson, Kaldor and Pasinetti. The main thrust of their critique of neoclassical models concerned the theory of capital, considered below. In addition, however, the neoclassical method of studying equilibrium was vehemently criticized. The main critic was perhaps Robinson, who argued that neoclassical models neglected what she called "historical time". Her own models, however, were not much better in that she concentrated her attention on *golden age growth*, a concept identical to the neoclassical steady state, where an economy is in equilibrium with a constant growth rate. Her approach did, however, have the merit of stressing the unrealistic nature of the concept.[19]

Common to the theories of Robinson, Kaldor and Pasinetti was the so called "Keynesian" theory of income distribution. There were two sources from which this theory sprang. One was the "widow's cruse" theory of profits contained in Keynes' *Treatise on Money*, taken up again in his *How to Pay for the War*.[20] The other source was Kalecki's theory of the trade cycle. Kalecki was a Polish economist who, as early as 1933, had derived a theory which had much in common with Keynes' *General Theory*.[21] An important aspect of Kalecki's work was that, as a result of his Marxist background, he concentrated on income distribution, something neglected in the *General Theory*. Kalecki argued that profits were proportional to the sum of capitalists' consumption and investment.[22]

The most widely known version of the Keynesian theory of distribution is, however, that of Kaldor (1956).[23] In deriving this theory Kaldor's starting point was that, in equilibrium, national income, which is identically equal to the sum of profits and wages, must equal consumption plus investment. Using self-explanatory notation,

$$P + W = C + I.$$

The simplest case is obtained by assuming that all wages are consumed, and that a fraction, s, of profits is saved, so that

$$C = (1 - s)P + W.$$

From these two equations it follows that

$$P/K = (1/s)\ I/K$$

where P/K is the rate of profit on capital and I/K the growth rate.[24] The rate of profit is determined by the growth rate and the propensity to save out of profits. The Keynesian theory asserts that it is investment that is exogenous, and that this determines the rate of profit.

Though this theory was used by its authors to provide an alternative to the marginal productivity theory of profits, the two are in no way contradictory. Savings behaviour and whether or not firms maximize profits,[25] the basis for marginal productivity theory, are separate issues.

Although attempts were made to destroy the theory, either by ridicule, or by criticizing its formulation,[26] it was a theory for which a defence could be provided. It might depend on assumptions which, from a neoclassical point of view, look peculiar, but that is a different matter.

25.2 THE THEORY OF CAPITAL

Robinson's complaints[27]

The post-war controversies over the theory of capital, to which Joan Robinson was the main contributor, are sometimes seen as a reaction against aggregative growth models. But though such growth models clearly fuelled the controversy, they were not its starting point, for Robinson's original article, "The production function and the theory of capital", which started the controversy, was published in 1953, the first neoclassical growth models appearing only in 1956. Robinson's target was rather marginal productivity theories of distribution of the Hicksian type.

Robinson opened her article with a direct attack on the neoclassical concept of the production function. Firstly, she argued that by emphasizing the role of factor substitution, it diverted attention from the more important issues of factor supply and technical change. More important, however, was her argument that the proponents of an aggregate production function had evaded the issue of how capital was to be measured.

> Moreover the production function has been a powerful instrument of miseducation. The student of economic theory is taught to write $O=f(L,C)$ where L is a quantity of labour, C is a quantity of capital and O a rate of output of commodities. He is instructed to assume all workers alike, and to measure L in man-hours of labour; he is told something about the index number problem involved in choosing a unit of output; and he is then hurried on to the next question, in the hope that he will forget to ask in what units C is measured. Before he ever does ask, he has become a professor, and so sloppy habits of thought are handed on from one generation to the next.[28]

In this article and in her subsequent writings[29] the neoclassical theory is attacked on three grounds.[30] It is important to keep these separate as they are completely separate arguments. (1) There is the Keynesian objection that demand may be insufficient to ensure full employment.[31] (2) Next there are Robinson's strictures against the use of the concept of equilibrium. She argued that equilibrium was not something an economy could ever get into: either an economy was, and always had been, in equilibrium, or else it would never be in equilibrium.[32] Her arguments here raised a variety of issues, concerning expectations, the age structure of the capital stock and the extent to which technical coefficients were fixed when capital goods were installed. (3) The most important argument, however, was that the production function could not be used *even to compare two equilibrium paths*, let alone to say anything about an economy out of equilibrium.

To understand this last point we have to consider the role of capital in the neoclassical account of capital accumulation.[33] The essence of the neoclassical story is that capital is accumulated by saving, the amount of consumption sacrificed being equal to the amount of capital created. This capital earns the marginal product of capital, which is the yield (interest) available to the saver. "Capital" here represents *two* things: the amount of saving undertaken, and the resulting change in the stock of goods (on which the marginal product of capital depends). Robinson's argument was that it was in general *impossible* to find a single measure of capital which could represent these two things. The reason why the value of the capital stock on which interest was calculated (the stock of savings) could not be the same as the physical capital stock was found in the so called price–Wicksell effect. This was the effect whereby any difference in the capital stock would result in a different rate of interest and hence a different relative price between consumption goods and capital.[34] At different interest rates the same sacrifice of consumption goods would produce different changes in the real capital stock.

In the 1950s and 1960s a typical response to this argument was to admit that Robinson was right in principle, but to treat the one-sector model, in which these problems are assumed away (there is a single commodity, usable either for consumption or as capital) served as a rough approximation to more realistic models. Consider Swan's defence of the one-sector model:

> From the idea of capital as a single stock there is in principle no sudden transition to the enormous who's who of all the goods in existence. Between the two extremes lies an ascending scale of nth-order dynamic systems, in which capital like everything else is more and more finely subdivided and dated, with ascending degrees of (potential) realism and (actual) complexity.[35]

A similar view was taken by Solow (1957) who, in defending econometric work using an aggregate production function, argued: "One can at least hope that the aggregate analysis gives some notion of the way a detailed analysis would lead."[36] He makes it clear, however, that he cannot provide a rigorous justification for this hope:

> let me make explicit that I would not try to justify what follows [the use of an aggregate production function] by calling on fancy theorems on aggregation and index numbers. Either this kind of aggregate analysis appeals or it doesn't. Personally I belong to both schools. If it does, I think one can draw some crude but useful conclusions from the results.[37]

Sraffa

Before considering some of the arguments which arose as a result of Robinson's claims, we have to consider another contribution which had a profound influence on the way the debate proceeded, namely Sraffa's *Production of Commodities by Means of Commodities* (1960). This was subtitled "Prelude to a Critique of Economic Theory", and was intended to revive

the Ricardian and Marxian approach to the problem of value and distribution. In contrast to the neoclassical tradition where commodity prices and the distribution of income are determined simultaneously, distribution is, in the Ricardian tradition as interpreted by Sraffa, determined *prior to* commodity prices: distribution is determined at the macroeconomic level, after which prices can be calculated from costs of production. The book's influence was threefold.

(1) Sraffa influenced the way in which the problem of capital was approached. His method, followed in the "reswitching" controversy which followed, was to consider a linear technology of the Leontief type. A technique of production was made up of a series of processes such as:

$$\begin{aligned} &4 \text{ tons corn} + 3 \text{ tons iron} + 2 \text{ units labour} \rightarrow 5 \text{ tons iron} \\ &2 \text{ tons corn} + 2 \text{ tons iron} + 1 \text{ unit labour} \rightarrow 3 \text{ tons wheat.} \end{aligned}$$

For such a technique it was possible to write down equations relating commodity prices, the wage rate and the rate of profit:

$$\begin{aligned} (4P^C + 3P^I + 2W)\,(1 + r) &= 5P^I \\ (2P^C + 2P^I + 1W)\,(1 + r) &= 3P^C. \end{aligned}$$

Bearing in mind that we are free to take one of these prices as *numéraire*, these equations can be solved to give a relation between the rate of profit and the real wage rate (in terms of the *numéraire*). It can be shown that there will be a negative relationship between the rate of profits and the real wage rate, the exact form of the relationship depending on the capital–labour ratios in the two sectors.

Although Sraffa was not the first to use this type of technology, his work led to the technological possibilities open to the economy being represented by a set of techniques of production, each technique having its own set of input–output coefficients. At any real wage rate, the technique actually used would be the one which yielded the highest rate of profit. From such a set of techniques it is possible to derive relationships between the wage rate, the rate of profit, output and capital per head.

(2) Sraffa's book revived interest, as Sraffa had intended, in the Ricardian/Marxian theory of value, relating the problem of capital measurement to the problems Ricardo and Marx had encountered with the theory of value.[38] One of the reasons for this was that Sraffa solved the technical problems underlying Ricardo's search for an invariable measure of value. He showed that prices could be measured in terms of the "standard commodity", an artificial commodity constructed so as to be produced with the economy's average capital–labour ratio. This invariable measure, however, could not do what Ricardo had wanted it to do, for whenever the technique of production changed, so too would the standard commodity itself.[39]

(3) Sraffa provided an alternative view of economics, one free from some of the alleged ideological implications of neoclassical theory.[40] The argument was that equations such as those above are insufficient to determine both the real wage rate and the rate of profit. Sraffa interpreted this to mean

that the distribution of income must be determined *outside* the system of pricing. One way is the Ricardian one of assuming the real wage rate to be fixed at subsistence; alternatively we might add a rate of exploitation (the ratio of profits to wages) in a Marxian manner. For Sraffa the details were not so important as the fact that it made it possible for him to argue that distribution and price determination were not simultaneous: distribution was logically prior to pricing, for once distribution is determined, the equations described above can be used to determine commodity prices. Sraffa saw this approach as radically different from the neoclassical.[41]

Reswitching

The issue which dominated discussions of capital theory in the 1960s was "reswitching". To understand this we must consider a conventional neoclassical production function with diminishing marginal products. Such a production function implies that as the rate of interest falls,[42] more capital-intensive techniques of production are introduced, and output per head rises. When the technology is analysed in terms of a set of Sraffian techniques, however, the possibility arises that as the rate of interest falls, the economy may, instead of moving towards a more capital-intensive technique, return to a less capital-intensive technique previously adopted at a lower rate of interest. In other words, as the rate of interest is reduced, the capital–output ratio, and possibly output per head as well, may first rise and then fall. This is the phenomenon known as "reswitching". Its importance was that it made it clear that the technology could be represented by a neoclassical production function only under very special circumstances.[43]

By the mid 1960s it became accepted that these arguments undermined the one-sector neoclassical model, the classic statement of this being that of Samuelson (1966b):

> The phenomenon of switching back at a very low interest rate to a set of techniques that had seemed viable only at a very high interest rate involves more than esoteric technicalities. It shows that the simple tale told by Jevons, Böhm-Bawerk, Wicksell and other neoclassical writers – alleging that, as the interest rate falls in consequence of abstention from present consumption in favour of future, technology must become in some sense more "roundabout", more "mechanized" and more "productive" – cannot be universally valid. ... There often turns out to be no unambiguous way of characterizing processes as "more capital intensive", more "mechanized" more "roundabout", except in the *ex post* tautological sense of being adopted at a lower interest rate and involving a higher real wage.[44]

Capital and general equilibrium

Controversy continued throughout the 1960s, however, for a variety of reasons, the main one being that the implications of reswitching for disaggregated neoclassical models were still not clear. Critics claimed that the whole of general equilibrium theory, based on supply and demand, was undermined, whilst others argued that only aggregative models were affected.[45]

From a methodological point of view, two responses to this situation are particularly interesting. One was to argue that the possibility of reswitching was an empirical issue.[46] Samuelson (1966b) doubted whether reswitching was very important,[47] whilst others investigated conditions necessary to rule out reswitching.[48] Most extreme of all was Ferguson's declaration that the whole issue was an empirical one, and that "Until the econometricians have the answers for us, placing reliance on neoclassical economic theory is a matter of faith."[49]

The other interesting response was that of Solow (1963), for he tried to re-instate the neoclassical vision through adopting a different approach, one which dispensed, so he claimed, with the need to measure capital. His approach was to ask what we needed to know about the technology in order to construct an optimal plan. Thus he argued that the crucial concept was not capital but the rate of return on investment: if a unit of consumption is sacrificed in one period, what return will it provide in terms of additional consumption in the following period. This was Fisher's rate of return.[50] Solow's argument was criticized by Pasinetti (1969), who argued that implicit in the Fisher approach was an "unobtrusive postulate", which amounted to *assuming* that capital intensity varied inversely with the rate of interest. The details of his argument are not so important as the reason why it attracted so much interest: it claimed to show that not only was the one-sector parable untenable, but so too were more general models. Solow's approach of diverting attention to the rate of return on investment did not lessen the controversy.

Intertemporal general equilibrium

The reason why the critics of neoclassical economics were so persistent is obvious: they believed that neoclassical theories of growth and distribution were fundamentally flawed. Why, however, did the proponents of neoclassical economics not simply ignore these criticisms? Why was reswitching seen as more than a minor puzzle? The answer would appear to be that it concerns a fundamental assumption made throughout the theory of competitive equilibrium: the assumption of diminishing marginal rates of substitution.[51] Without diminishing marginal rates of substitution there is no assurance that the familiar tangency conditions will yield a maximum rather than a minimum of profits. To emphasize the importance of the idea, note that a similar assumption has to be made about utility functions, otherwise the tangency of an indifference curve with a budget line would imply minimization, not maximization, of utility. The assumption of diminishing marginal rates of substitution is as fundamental to the theory of competitive equilibrium as is the assumption of rational behaviour (maximization of profits or utility).

An implication of diminishing marginal rates of substitution is that the price system is "connected": that if an allocation of resources is optimal at two sets of prices, it will be optimal at all intermediate sets of prices. To see this, consider an isoquant of the type often found in linear programming:

one with a series of corners. The corner points are optimal at a range of factor price ratios: for example, in Figure 25.2, B will be an optimum provided that the iso-cost lines are steeper than BC and flatter than AB. The point is that if a point, B, is optimal at two factor price ratios, then it is also optimal at any intermediate factor price ratios.[52] Reswitching, however, implies that an allocation of resources may be optimal at two sets of prices, but not at intermediate sets of prices. In other words, that the price system is not connected. Thus reswitching raised doubts about a fundamental assumption: about part of the "hard core" of neoclassical economics.

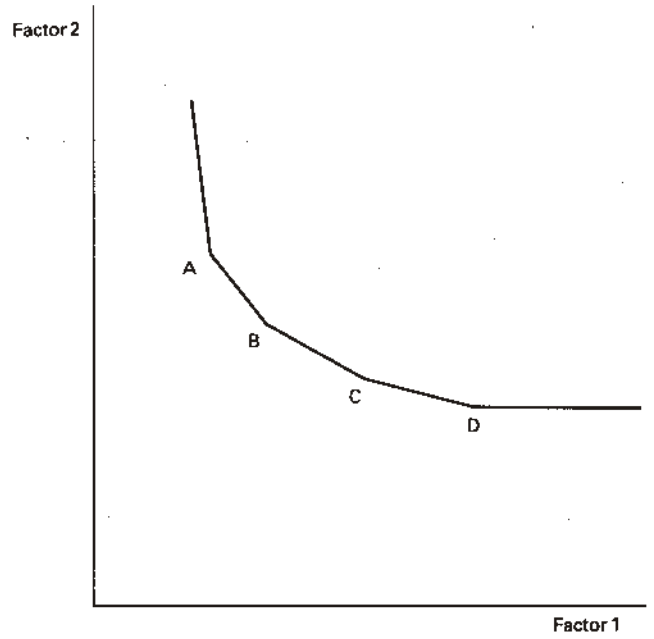

FIGURE 25.2 *An Isoquant*

A resolution of this paradox was provided in Bliss's *Capital Theory and the Distribution of Income* (1975).[53] The key to this book is that capital theory is discussed in the context of an explicit model of intertemporal general equilibrium. This distinguishes goods not only in terms of their physical characteristics, but also in terms of their date of delivery. Thus if we have n types of good, and T time periods, we have nT dated "commodities". If there is a market for each dated commodity, we have nT prices. Denote the price of the ith commodity in period t as $p_{i,t}$. Implicit in these prices is a set of interest rates. For example, the interest rate on good i at time t, $r_{i,t}$, is given by $(p_{1i,t} - p_{i,t-1})/p_{i,t-1}$. This is the yield to be obtained from holding good i for one period at time t.

Reswitching is paradoxical because it implies that the set of prices at which a particular technique is chosen is not connected: that a technique is optimal at two sets of prices, but not at sets of prices in between. Bliss, using this intertemporal general equilibrium framework, was able to show that this was an illusion.

Bliss used this intertemporal general equilibrium framework to argue that the paradox of reswitching arose because economists were in the habit of assuming only a limited set of prices: economists werein the habit of considering only those sets of prices in which the rate of interest was constant. If we broaden the set of prices we consider, so that we allow for price paths on which the rate of interest is changing, then the paradox disappears. This rather abstract idea is best understood with the aid of a simple example. Assume that there is a single commodity, and three time periods. Suppose that there is reswitching in the sense that a given technique is optimal at rates of interest of 10% and 20%, but that some other technique is chosen if the interest rate is 15%. The question is whether this implies that the price system is not connected?

Take the price of the commodity in period 1 as numeraire. If the rate of interest is 10%, the price must be 1.1 in period 2, and 1.21 in period 3. If the rate of interest is 20%, the corresponding prices are 1.2 and 1.44. These prices are shown in Table 25.1. If the price system is connected, the

Table 25.1

	A $r = 10\%$	B $r = 20\%$	C
1	1	1	1
2	1.1	1.2	1.15
3	1.21	1.44	1.325

technique we are considering must be optimal at an average of these two sets of prices. Such an average is shown in column 3 of the table. For example, the price in period 2 is $(1.1 + 1.2)/2 = 1.15$. The important thing to note about column 3 is that the rate of interest is *not* 15%: it is 15% from periods 1 to 2, but only 14% from periods 2 to 3.

By showing that a technique may be optimal at 10% and at 20%, but not at 15%, reswitching suggests that the price system is not connected. This is, however, an "optical illusion", for if we allow for price paths with a variable rate of interest, the price system appears completely connected.[54] The paradox disappears.

25.3 CONCLUSIONS

For all the intellectual effort that went into growth and capital theory in the 1950s and 1960s, it can be argued that little light was shed on the factors causing some countries to grow faster than others. The origins of modern growth theory lay with Harrod and Domar, who were concerned about the possibility of secular stagnation. From the mid-1950s growth theory came to be incorporated within general equilibrium theory, and it attracted attention as the theory of dynamic equilibrium. From the early 1970s interest in growth theory lessened. Why?

(1) Growth theory ceased to be topical: the problems of inflation, unemployment and exhaustible resources came to be seen as more important than the problem of explaining the conditions under which steady growth might occur. In addition, the concept of rational expectations opened up new areas of theoretical research outside growth theory.

(2) More importantly, however, the main results obtainable within the framework of neoclassical growth theory had, by the early 1970s, been worked out. Growth theory was understood, and ceased to be a separate subject. Models of dynamic equilibrium could be used routinely by economists whose main concerns lay in other fields, such as public finance or monetary economics.

(3) Though many economists continued to defend the general neoclassical model, the Cambridge critique of the neoclassical production function did serve to emphasize that the one-sector neoclassical model could be no more than a parable. It also undermined the attempt to provide an empirical basis for growth theory through estimating aggregate production functions. Thus though the Cambridge critique was not the main reason for the loss of interest in growth theory, it was a contributory factor.

With the theory of capital, controversy was stimulated by economists who believed that the neoclassical research programme, based on maximization and marginal productivity, was fundamentally flawed, and in need of replacement. To explain the persistence of the controversy, however, we need to explain why neoclassical economists could not simply ignore the criticisms, as they ignore so many others. The reason would seem to be that reswitching the "paradox" of reswitching cast doubt on the assumption of convexity, fundamental to so much of mainstream economic theory. It was because it cast doubt on such a fundamental postulate that dismissing reswitching as "unlikely" was beside the point.

It is because of this that Bliss's defence of the neoclassical theory, showing that reswitching does not imply any non-convexity at all, is so important. It is important, however, that this defence of neoclassical theory was achieved only at the cost of making it even clearer that the economy under discussion could not possibly describe any actual economy. With an aggregative neoclassical growth model it is possible to imagine that it could be extended to cope with the presence of uncertainty. With Bliss's intertemporal general equilibrium model it is clear that the introduction of time and uncertainty would involve radical changes in the theory.