‘Why Disinflation through Currency Pegging may Cause a Boom: the Role of Forward-Looking Wage Setting’ by John Fender and Neil Rankin (October 2008 version)

TECHNICAL APPENDIX (NOT FOR PUBLICATION)

The aim of the Technical Appendix is to provide derivations of some of the results in the paper that the authors consider inappropriate for inclusion in the main paper (for considerations of length and/or because the derivations are of no intrinsic interest).

## 2. Structure of the Model

*Derivation of the First-order Conditions* (*Equations* (*12*) – (*14*)):

The Lagrangean for the maximisation of (8) subject to (9) – (11) is

$$\Lambda \equiv \sum_{t=0}^{\infty} \beta^t \left[ \delta \ln C_{jt} + (1-\delta)\ln(M_{jt}/P_t) - \eta[(W_t/W_{jt})^{\varepsilon} N_t]^{\zeta} \right]$$
$$+\sum_{t=0}^{\infty} \lambda_t \left[ M_{jt-1} + I_{t-1}B_{jt-1} + W_t^{\varepsilon} N_t W_{jt}^{1-\varepsilon} + \Pi_t + S_t - P_t C_{jt} - M_{jt} - B_{jt} \right]$$
$$+\sum_{t=0}^{\infty} \mu_t \left\{ (-1)^t/2 + 1/2 \right\} \left[ W_{jt} - W_{jt+1} \right].$$

The term {.} imposes the constraint that the wage is re-set every two periods. The first-order conditions are as follows:

(Maximising with respect to $B_{jt}$): $\quad -\lambda_t + \lambda_{t+1} I_t = 0,$ (T1)

(Maximising with respect to $C_{jt}$): $\quad \delta\beta^t / C_{jt} = \lambda_t P_t,$ (T2)

Combining (T2) with the equivalent expression for $C_{jt+1,}$ and using (T1), we obtain (12).

(Maximising with respect to $M_{jt}$):

$$\beta^t (1-\delta)/M_{jt} - \lambda_t + \lambda_{t+1} = 0. \qquad \text{(T3)}$$

Combining this with (T1) and (T2) gives (13).

(Maximising with respect to $W_{jt}$):

$$\beta^t \varepsilon\eta\zeta L_{jt}^{\zeta}/W_{jt} + \lambda_t (1-\varepsilon) L_{jt} - \mu_{t-1}\{(-1)^{t-1}/2 + 1/2\} + \mu_t \{(-1)^t/2 + 1/2\} = 0 \qquad \text{(T4)}$$

Using (T2) to eliminate $\lambda_t$ from (T4) and writing out separate versions for even and odd $t$, we have:

$$\mu_t = \beta^t(\varepsilon-1)\delta L_{jt} / P_t C_{jt} - \beta^t \varepsilon\eta\zeta L_{jt}^{\zeta} / W_{jt} \qquad \text{if } t \text{ is even,} \tag{T5}$$

$$\mu_{t-1} = -\beta^t(\varepsilon-1)\delta L_{jt} / P_t C_{jt} + \beta^t \varepsilon\eta\zeta L_{jt}^{\zeta} / W_{jt} \qquad \text{if } t \text{ is odd.} \tag{T6}$$

In period $t+1$, we have:

$$\mu_t = -\beta^{t+1}(\varepsilon-1)\delta L_{jt+1} / P_{t+1} C_{jt+1} + \beta^{t+1} \varepsilon\eta\zeta L_{jt+1}^{\zeta} / W_{jt+1} \qquad \text{if } t \text{ is even.} \tag{T7}$$

We can now combine (T5) and (T7) to eliminate $\mu_t$. Note that, from the constraint (11), $W_{jt} = W_{jt+1}$ ($= X_t$) when $t$ is even. The resulting equation can therefore be solved for $W_{jt}$, which yields (14).

## 3. General Equilibrium

Many of the derivations of the loglinearised equations are straightforward and not reported here.

*Derivation of Equation (25)*:

From equation (16) we have

$$Z_t = \frac{1-\delta}{\delta}\frac{I_t}{I_t - 1}, \text{where } Z_t \equiv M_t / P_t C_t. \tag{T8}$$

Therefore

$$\frac{dZ_t}{Z_t} = \frac{-1}{(I_t - 1)}\frac{dI_t}{I_t}.$$

But, from (12), the steady-state interest rate $I = 1/\beta$, so equating $I_t$ to this, (25) follows.

*Derivation of Equation (26)*:

From (12), since $C_{jt} = \Omega_{jt}/P_t$:

$$\frac{\Omega_{t+1}}{M_{t+1}} = \beta I_t \frac{\Omega_t}{M_t}\frac{M_t}{M_{t+1}}. \tag{T9}$$

Using the definition $Z_t = M_t / \Omega_t$ differentiating and using (22), we obtain

$$\mu_{t+1} = i_t + (z_{t+1} - z_t)\,.$$

Substituting (25) to eliminate $i_t$ gives (26).

*Derivation of Intertemporal Balance of Trade Condition* (*Equation* (*33*)):

As the LHS of (19) for the reference steady state, $-I_R B_R$, is zero, the approximation is about a steady state with a zero trade balance. Changes in the price of traded goods and in the interest rate therefore drop out of the log-linearised version of (19).

*Derivation of New Wage Equation* (*Equation* (*34*)):

Differentiating (20) gives:

$$\frac{dX_t}{X} = \frac{\varepsilon\eta\zeta X^{-1-\varepsilon(\zeta-1)}}{(\varepsilon-1)\delta\{1+\varepsilon(\zeta-1)\}}\left\{\frac{A}{(1+\beta)W^{\varepsilon}N/\Omega} - \frac{(1+\beta)W^{\varepsilon\zeta}N^{\zeta}B}{\{(1+\beta)W^{\varepsilon}N/\Omega\}^2}\right\} \qquad \text{(T10)}$$

where $A \equiv \varepsilon\zeta W_t^{\varepsilon\zeta-1}N_t dW_t + \zeta W_t^{\varepsilon\zeta}N_t^{\zeta-1}dN_t + \beta\varepsilon\zeta W_{t+1}^{\varepsilon\zeta-1}N_{t+1}^{\zeta}dW_{t+1} + \beta\zeta W_{t+1}^{\varepsilon\zeta}N_{t+1}^{\zeta-1}dN_{t+1}$.

and $B \equiv \varepsilon W_t^{\varepsilon-1}N_t\Omega_t^{-1}dW_t + W_t^{\varepsilon}\Omega^{-1}dN_t + \beta\varepsilon W_{t+1}^{\varepsilon-1}N_{t+1}\Omega_{t+1}^{-1}dW_{t+1} + \beta W_{t+1}^{\varepsilon}\Omega_{t+1}^{-1}dN_{t+1}$
$-W_t^{\varepsilon}N_t\Omega^{-2}d\Omega_t - \beta W_{t+1}^{\varepsilon}N_t\Omega_{t+1}^{-2}d\Omega_{t+1}$.

Recognising that log-linearisation is about the ZISS and using lower-case letters to denote log deviations and rearranging, $A$ becomes:

$$A \equiv \zeta W^{\varepsilon\zeta}N^{\zeta}\{\varepsilon w_t + n_t + \beta\varepsilon w_{t+1} + \beta n_{t+1}\}.$$

Likewise, $B$ can be rewritten as

$$B \equiv W^{\varepsilon}N\Omega^{-1}\{\varepsilon w_t + n_t + \beta\varepsilon w_{t+1} + \beta n_{t+1} - \omega_t - \beta\omega_{t+1}\}.$$

Applying the (zero inflation) steady-state condition to (20), we obtain

$$X = \left\{\frac{\varepsilon\eta\zeta}{(\varepsilon-1)\delta}W^{\varepsilon(\zeta-1)}N^{\zeta-1}\Omega\right\}^{1/\{1+\varepsilon(\zeta-1)\}}.$$

Substituting for $A$, $B$ and $X$ in (T10), manipulating and rearranging, we obtain (34).

*Derivation of Second-Order Difference Equation in the New Wage* (*Equation* (*44*)):

Using (36) and (48), we obtain

$$n_t = \omega_t - w_t. \tag{T11}$$

Substituting this and the equivalent expression for $n_{t+1}$ into (34) and then using (35) and the equivalent expression for $w_{t+1}$ to eliminate $w_t$ and $w_{t+1}$ gives, after some manipulation, and using the definition $\gamma \equiv \zeta / [1 + \varepsilon(\zeta - 1)]$, we obtain equation (44).

*Derivation of the Stable Eigenvalue* (*Equation* (*45*)):

Let $x_t = A\nu^t$ be a solution to the homogeneous part of (44) – i.e., it equates the LHS to zero. Substitution and some manipulation yield the following quadratic equation:

$$\beta(1-\gamma)\nu^2 - (1+\beta)(1+\gamma)\nu + (1-\gamma) = 0.$$

$\lambda$, given by (45), is the smaller of the two solutions of this equation. See below for a proof that $0 < \lambda < 1$.

*Derivation of Constant-Inflation Steady-State Level of Output* (*Equation* (*46*)):

Assume the economy is in a constant-inflation steady state at rate $\mu$, which means that all nominal variables grow at that rate, so $x_{t-1} = x_t - \mu$, $x_{t+1} = x_t + \mu$ and $\omega_{t+1} = \omega_t + \mu$, then substituting in (44) and solving gives:

$$x_t = \omega_t + \frac{\mu}{2} + \frac{\mu(\beta-1)}{2\gamma(1+\beta)}. \tag{T12}$$

Using (35) we can derive an expression for the wage:

$$w_t = \omega_t + \frac{\mu(\beta-1)}{2\gamma(1+\beta)}. \tag{T13}$$

Using (48) generates (46).

## Appendix

*Proof that the Eigenvalues Satisfy* $0 < \lambda < 1$, $\lambda' > 1$:

In the Appendix, part (iii), we saw that the characteristic equation could be written as $F(\nu) = 0$, where $F(\nu)$ is a quadratic function whose graph is therefore a parabola. Evaluating this function at $\nu = 0$ and $\nu = 1$, we have:

$$F(0) = \phi, \quad F(1) = (1+\beta)(\phi - 1).$$

Since we noted that $0 < \phi < 1$, it follows that $F(0) > 0$, $F(1) < 0$. The graph of the parabola must therefore intersect the horizontal axis once in the interval (0,1) (which yields the stable eigenvalue, $\lambda$), and once in the interval $(1,\infty)$ (which yields the unstable eigenvalue, $\lambda'$).

*Derivation of the Constant-Inflation Steady-State Value of* $s_t$:

Using the fact (see (43)) that $e_t = \omega_t$, $s_t$ ($\equiv x_t - e_t$) is equivalently $x_t - \omega_t$. An expression for the steady-state value of $x_t - \omega_t$ when the inflation rate is $\mu$ can easily be derived from (T13).

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