

# Mathematical Modelling for Economists

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(slides preliminary until end of term)

# Contents

- 1 Introduction
- 2 Good questions and good theories
- 3 Examples from the literature

# How much does monopoly cost?

< 0 %	0 – 5 %	5 – 10%	10 – 25%	25 – 50%	> 50%	can't say
1	2	6	20	12	0	1

(Don't worry if you've not studied A-level economics: the new ideas here will take a few minutes to understand; the real work lies in thinking about them, which you can do too.)

## Starting to think methodologically: 1920s US data

- have sectoral-level data for c. 45% of manufacturing firms on
  - ① excess profits (relative to the average)
  - ② sales revenue
- **assume:** the 20's were stable  $\Rightarrow (AC \approx \text{constant})$ ,  $\Rightarrow (MC \approx AC)$   
(why, and why make these assumptions?)
- **assume:** the **price elasticity of demand**, is unity:

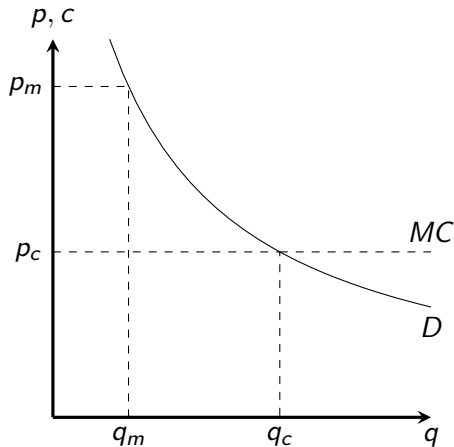
$$\eta \equiv \left| \frac{dq}{dp} \frac{p}{q} \right| \approx 1$$

(always be suspicious of round numbers' convenience, but for now ...)

- can we fill in the final column? Why isn't it just  $\pi^+ \equiv \text{excess } \pi$ ?

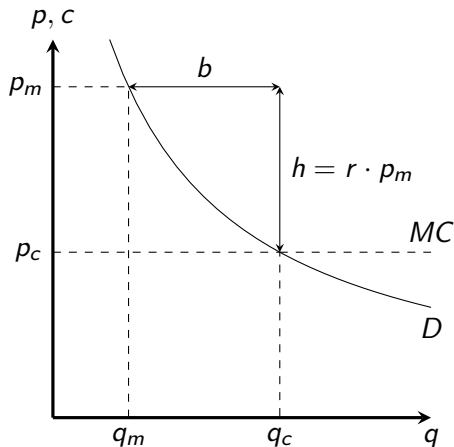
Industry	return on $K$	$\pi^+$ (mn)	$r \equiv \frac{\pi^+}{\text{sales rev}}$	welfare cost
Bakery products	17.5%	\$17	5.3%	
Flour	11.9%	\$1	0.4%	
Confectionary	17.0%	\$7	6.1%	

# A graphical approach



- why is  $MC$  curve flat?
  - 1 double-check your answer: derive  $MC$ ,  $AC$  from  $TC$
  - 2 intuition for the result?
- what object(s) do we want to calculate?
- can we use our data to do so?

# Using our data to derive $b$ and $h$



- using slope and elasticity's definitions:

$$-b \approx \frac{dq}{dp} h$$

$$-b \frac{p_m}{q_m} \approx \frac{dq}{dp} \frac{p_m}{q_m} h$$

$$b \approx \eta h \frac{q_m}{p_m}$$

$$\approx \eta (r \cdot p_m) \frac{q_m}{p_m}$$

$$\approx \eta r q_m$$

$$DWL \approx \frac{1}{2} b h \approx \frac{1}{2} r^2 p_m q_m \eta$$

- what are  $p_m, q_m$ ? Can we use this representation?

## Plugging in the numbers

- Bakery:

$$\begin{aligned} DWL_B &\approx \frac{1}{2} \cdot r^2 \cdot (p_m \cdot q_m) \cdot \eta = \frac{1}{2} \cdot r^2 \cdot \frac{\pi^+}{r} \cdot \eta \\ &= \frac{1}{2} \cdot \left( \frac{5.3}{100} \right) \cdot 17 \cdot 1 \approx \$0.451\text{mn} \end{aligned}$$

- summing up across the sectors yields \$26.5mn
- scaling up to the whole manufacturing sector yields \$US58.9mn
- thus, monopoly cost US manufacturing about  $\frac{1}{13}$ % of US GDP in the 1920s

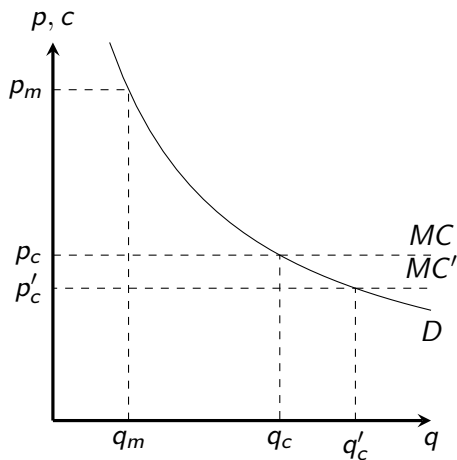
# Thinking about the results

- revisiting the assumptions
  - what if we integrate, rather than assuming a triangle?
  - was the US economy of the 1920s significantly different from today's?
  - risk-weighted returns more appropriate?
  - how treat intermediate industries?
  - how account for price effects?
  - what percentage of excess profits are due to monopoly?
- policy implications
  - can show similar small welfare losses in other areas (taxes, tariffs, etc.)
  - thus, does inefficient government policy matter?
- just plain understanding
  - how is  $\pi^+$  calculated? (Examples of the importance of understanding where your numbers come from: [here](#) and [here](#))



# Questioning the model

*The best of all monopoly profits is a quiet life. (John Hicks)*



- my goal: lure you into a detailed set of calculations
- how much does the competitive marginal cost have to drop to double welfare losses?
- see [Wikipedia entry](#) for more critiques

## References & c.

- original analysis in Harberger (1954)
  - Eric Rasmussen's G604 History of Thought slides print the triangular formula
- Leibenstein (1966) documents examples of monopolies that are productively inefficient
- Perelman (2011) reviews the debate and issues arising
- next lecture, Solow (1997), Friedman (1953) and at least section VIII of Stigler (1950b) (the full article and Stigler (1950a) give the full background)
- nice historical twist: Caliendo and Parro estimated the welfare effects of NAFTA to be c. 0.08% ...

# How did Economics get that way ... (Solow, 1997)?

- prose, classifications replaced by diagrams, data, tables, equations
- compare Flux (1903) to Lommerud, Meland, and Sjørgard (2003)
- 'civilised' replaced by technical (hence 'autism' concern)
- not terribly formal, in mathematical sense
- dominated by model building & testing, due to data availability?

*In the last five hundred years we have had five major concept-driven revolutions ... [and] about twenty tool-driven revolutions ... The effect of a concept-driven revolution is to explain old things in new ways. The effect of a tool-driven revolution is to discover new things that have to be explained. ... We have been more successful in discovering new things than in explaining old ones. (Dyson, 2005)*

- economists mostly don't think about methodology
  - One of the best pieces of advice Rudi Dornbusch gave me was: never talk about methodology; just do it. (Blanchard, 2018)*
- prefers biology's observations to theoretical physics' rigour (q.v. Lo and Mueller (2010))

## Should models be realistic?

*in general, **the more significant the theory, the more unrealistic the assumptions** . . . A hypothesis is important if it 'explains' much by little, that is, if it abstracts the common and crucial elements from the mass of complex and detailed circumstances surrounding the phenomena to be explained and permits valid predictions on the basis of them alone. To be important, therefore, a hypothesis must be descriptively false in its assumptions. (Friedman, 1953)*

*A well-designed model is, after all, a **judiciously chosen set of lies**, or perhaps more accurately put, **partial truths about reality**, which have been chosen so as to permit us to reason more effectively about some issue than we otherwise could. The model **must** be an oversimplification if it is to be tractable analytically. Optimality in model construction must be based on the **trade-off between these two desiderata** – accuracy of representation of reality and usability in analysis. (Baumol, 1992)*

# Why model at all?

*A model is a description of reality. So if it doesn't reflect reality, then it's not going to work. If you think the model error is basically second-order and it's not, then the terms you neglected are going to come to the fore and the model will fail. That doesn't mean you're going to do any better with intuition – presumably you used your intuition in picking the model and intuition can fail, too. (Myron Scholes, 2011)*

## Modelling gone wrong: LTCM?

*LTCM, which had made enormous profits trading on the basis of mathematical models, was on the verge of collapse. . . .*

*Before founding LTCM, Meriwether had run a massive trading operation at Salomon and had done very well over a long period. He had some of the top minds in finance working with him at LTCM, including Nobel Prize winners Robert Merton and Myron Scholes. I was amazed that they had done what it seemed they had, **betting the ranch on the basis of mathematical models.***

*Models can be a useful way of looking at markets and can provide useful input into making decisions. But **ultimately traders have to make judgments, because reality is always far messier and more complicated than models can capture.** (Robert Rubin, 11 November 2003, Financial Times)*

*the rout cost Medallion . . . around one-fifth of the fund – in a matter of days. . . . They were on the verge of capitulating when the market rebounded; over the remainder of the year, Medallion made up the losses and more, ending 2007 with an 85.9 percent gain. The Renaissance executives had learned an important lesson: **Don't mess with the models.** (Katherine Burton, 21 November 2016, Bloomberg)*

## Modelling gone right: the 3G spectrum auctions?

*Six European countries auctioned off spectrum licenses for “third-generation” mobile phones in 2000. In Germany and the United Kingdom, the spectrum sold for over 600 euros per person (\$80 billion in all, or over 2 percent of GDP). But in Austria, the Netherlands, Italy and Switzerland, the revenues were just 100, 170, 240 and 20 euros per person, respectively. To be sure, investors became more skeptical about the underlying value of the spectrum during 2000 (and they are even more skeptical today). But this is just a fraction of the story. The Netherlands auction was sandwiched between the U.K. and German auctions, and analysts and government officials predicted revenues in excess of 400 euros per person from the Italian and Swiss auctions just a few days before they began. **These other auctions were fiascoes primarily because they were poorly designed.** (Klemperer, 2002)*

See his [website](#) and [online book](#) for more information. See Shin (2010, pp.91-) for an explanation in terms of bubbles rather than design.

# Critiques of modelling

- ① models are too simple to capture reality
  - Friedman (1953): “The ideal types are not intended to be descriptive; they are designed to isolate the features that are crucial for a particular problem”
  - after modeling, can use judgment on the residual
    - ① e.g. monetary policy committees: “economics is the grammar of arguments about policy, not the policy” (Hahn, 1973)
    - ② Black-Litterman investment optimisation models
- ② common sense and intuition are better
  - ‘intuition’ reflect brains’ **mental models**
  - prefer a black box or a transparent model?
  - see, for example, **this** on the current ‘explainable AI’ debate
- ③ modelling often complicates the obvious

*Like mathematical theory, mathiness uses a mixture of words and symbols, but ... leaves ample room for slippage between statements in natural versus formal language ... [Mathiness] will be worth little, but cheap to produce, so it might survive as entertainment. (Romer, 2015)*

  - ideally, modelling aids transparency, clarity, and may surprise us



# Homework

- 1 will pick groups by Friday 8/02/19
  - need **7 – 8** members / group for a total of 21 groups
  - ideas?
  - group members to sign up on Canvas
- 2 section VIII of Stigler (1950b) discusses criteria satisfied by a good theory; if interested, the full article and Stigler (1950a) give the full background
- 3 Samuelson (2005, §2) discusses the evolution of a theory, and the interplay between theory and empirical evidence
- 4 Gilboa et al. (2014) see models – even when ‘wrong’ – as providing cases, in the same way that data does

## Practical or policy relevance

*The most important decisions a scholar makes are what problems to work on. (James Tobin)*

- will the newspapers be interested?
- theory may start here, but it can also take a long time to offer much
- the most highly cited papers often are policy relevant rather than deeply theoretical
  - many economists think that the main problem of our research is that it is written for no one
- **positive** questions: what **is** climate change likely to do?
- **normative** questions: what **should** be done about it?

# Ease of explanation

- does it pass the roommate/parent test?
- when posing questions, a one sentence description is often excellent:
  - ①  $x^n + y^n = z^n$  has no non-zero integer solutions for  $x, y$  and  $z$  when  $n > 2$
  - ② every even number greater than four can be written as the sum of two odd prime numbers
  - ③ the optimal corporate tax rate is zero
  - ④ how does extraction from a commons change when the agents have access to capital markets?

# Personal commitment

*I have had my results for a long time: but I do not yet know how I am to arrive at them. (Carl Friedrich Gauss)*

- do you care?
- answer unknown to the researcher: you want to know the answer, or explore the consequences of an idea
  - a lot of good researchers are 'committed' to their ideas - q.v. Dirac
  - what assumptions does it take to get the results we expect?

# Fruitfulness

*Truth will sooner come out from error than from confusion. (Francis Bacon, Novum Organum)*

*any hypothesis, however absurd, may be useful in science, if it enables a discoverer to conceive things in a new way ... when it has served this purpose by luck, it is likely to become an obstacle to further advance. (Russell, 1945)*

*the relevant question to ask about the 'assumptions' of a theory is not whether they are descriptively 'realistic', for they never are, but whether they are sufficiently good approximations for the purpose in hand. And this question can be answered only by seeing whether the theory ... yields sufficiently accurate predictions (Friedman, 1953)*

# Internal coherence / consistency

- are any of the theories' axioms or implications contradictory?
- very basic criterion (thus, not even mentioned by Stigler (1950b))
- not always easy to assess
  - 1 most famous
    - Russell's discovery of an inconsistency in Frege's foundations
    - consequence: Gödel's incompleteness theorem (Nagel and Newman, 1959)
  - 2 ranking sets of objects
    - Bossert, Pattanaik, and Xu (2000) characterise min-max preferences in terms of four axioms, including independence
    - Arlegi (2003): shows min-max inconsistent with independence axiom
    - Geist and Endriss (2011): the (Bossert, Pattanaik, and Xu, 2000) axioms are inconsistent: can't characterise any set preferences

# Generality

*The successful theory was always more general than the theory it supplanted (Stigler, 1950b)*

What does **general** mean?

- 1 Ockham's razor (parsimony): shave away unnecessary assumptions
  - **strong form**: if two theories predict the same results, use the simplest
  - **weak form**: when first posing a question, simplify it to a 'toy example'
- 2 explain more phenomena: add variables

*always easy and usually sterile to introduce a new variable into a system, which then becomes more general (Stigler, 1950b)*

*The theory of imperfect or monopolistic competition ... is an attempt to construct such a more general theory. Unfortunately, it possesses none of the attributes that would make it a **truly useful** general theory. (Friedman, 1953)*

Stigler's critique: Leibenstein doesn't have a general theory (Perelman, 2011)

# Manageability / tractability

*economists tacitly agreed that it is better to have a poor, useful theory than a rich, useless one . . . Manageability should mean the ability to bring the theory to bear on specific economic problems, not ease of manipulation. The economist has no right to expect of the universe he explores that its laws are discoverable by the indolent and the unlearned. (Stigler, 1950b)*

- what does bringing a theory to bear on specific economic problems mean?
  - 1 theory uses variables that seem right (hence concerns about principal components analysis)
  - 2 theory uses operationalisable variables (e.g. concerns about human-capital-loss parameter (Ljungqvist and Sargent, 2008) or Solow residual)



# External validity

*[Macroeconomics] has become so mesmerized with its own internal logic that it has begun to confuse the precision it has achieved about its own world with the precision that it has about the real one. ... macroeconomic research has been in “fine-tuning” mode ... when we should be in “broad-exploration” mode. (Caballero, 2010)*

- Stigler's congruence with reality
- theories must “explain” “facts”
  - 1 consistency with ‘stylised facts’
  - 2 formal econometrics
  - 3 enough to get comparative statics right? (Samuelson, 2005)
- Samuelson (2005): what if the facts seem to oppose a theory?
  - 1 question the facts: q.v. Palacios-Huerta and Volij (2009) v. Levitt, List, and Sadoff (2011)
  - 2 look for missing factors in the theory
  - 3 how close does the theory have to be?
  - 4 is the deviation important in utility terms (q.v. Rosenbrock's banana)?
  - 5 can the theory be re-interpreted as the end result of learning?

## Falsifiability: evidence underdetermines theory I

*Factual evidence can never 'prove' a hypothesis; it can only fail to disprove it, ... Observed facts are necessarily finite in number; possible hypotheses, infinite. If there is one hypothesis that is consistent with available evidence, there are always an infinite number that are. (Friedman, 1953)*

*Theories are ... never empirically verifiable. ... it must be possible for an empirical scientific system to be refuted by experience (Popper, 1959)*

Example (Continue the sequence: 0, 1, 2, ...)

2 (A000720, starting at  $n = 1$ )? 3 (A007953)? 10 (A007089)? 720! (see notes under A000197)? ...?

Frustration: are there only two sequences beginning (with  $n = 0$ ) 1, 0, 0, 0, 0, 0, 1, 0, 2, 1, 1 ...?s ([This link](#) is fun.)

## Falsifiability: evidence underdetermines theory II

- good theories must offer the possibility of being proven wrong
  - ① “tax cuts can spur economic growth”
  - ② “tax cuts cannot spur economic growth”
- worst than being falsified: “not even wrong” (Pauli)
- widely misunderstood: e.g. can statistical inference **accept** a hypothesis?

*Not only were such specific implications not sought and tested, but there was a tendency, when there appeared to be the threat of an empirical test, to reformulate the theory to make the test ineffective. (Stigler, 1950b)*

- a **website** on spurious correlations; a **talk** on doing this in big data

## Aesthetic qualities?

*Beauty is the first test: there is no permanent place in this world for ugly mathematics. (Godfrey H. Hardy)*

*It is more important to have beauty in one's equations than to have them fit experiment... If one is working from the point of view of getting beauty in one's equations, and if one has really a sound insight, one is on a sure line of progress. If there is not complete agreement between the results of one's work and experiment, one should not allow oneself to be too discouraged, because the discrepancy may well be due to minor features that are not properly taken into account and that will get cleared up with further development of the theory. (Paul Adrien Maurice Dirac)*

*It is possible to know when you are right way ahead of checking all the consequences. You can recognize truth by its beauty and simplicity. (Richard Feynman)*

# Adam in the Garden of Eden

Letting  $\mathbf{a} \equiv (a_1, \dots, a_T)$  be Adam's **endowment stream** of apples, assume:

- ① Adam expects to live no longer than  $T = 120 \times 365$  days
- ② apples picked at date  $t$  must be consumed at date  $t$
- ③ Adam cares only about the stream of apples that he consumes,  $\mathbf{c} = (c_1, \dots, c_T)$
- ④ on any given day
  - (T.1) 2 apples  $\succ$  1 apple  $\succ$  0 apples
  - (T.2) 2 apples  $\succ$  more than 2 apples
- ⑤ across days: for any  $t < T$ 
  - (A.1)  $(c_1, \dots, c_{t-1}, \mathbf{1}, c_{t+1} - 2, c_{t+2}, \dots, c_T) \succ (c_1, \dots, c_{t-1}, \mathbf{0}, c_{t+1}, c_{t+2}, \dots, c_T)$
  - (A.2)  $(c_1, \dots, c_{t-1}, \mathbf{2}, c_{t+1} - 1, c_{t+2}, \dots, c_T) \succ (c_1, \dots, c_{t-1}, \mathbf{1}, c_{t+1}, c_{t+2}, \dots, c_T)$

# Adam's first traumatic experience

Denote by  $\langle x_1, \dots, x_K \rangle \equiv (x_1, \dots, x_K, 0, \dots, 0)$ .

## Theorem

Let  $a_1^* = \dots = a_{17}^* = 0$  and  $a_t^* = 1 \forall t \in \{18, \dots, T\}$ . Then, under the above assumptions,  $\langle 1 \rangle \succ \mathbf{a}^*$ .

Discursion: what is a theorem?

- ① “Given assumptions  $X$ , it follows that  $Y$ ”
- ② existence theorems: “Given assumptions  $X$ , there **exists** an  $Y$ ”
- ③ uniqueness theorems: “Given assumptions  $X$ , there exists **only one**  $Y$ ”
- ④ sufficient conditions: “**If**  $X$  then  $Y$ ”
- ⑤ necessary conditions: “**Only if**  $X$  then  $Y$ ”
- ⑥ necessary and sufficient conditions: “**If and only if**  $X$  then  $Y$ ”

## Adam's first traumatic experience

### Lemma

$$\langle 1, 0, 0 \rangle \succ \langle 0, 1, 1 \rangle$$

### Proof.

$\langle 0, 2, 0 \rangle \succ \langle 0, 1, 1 \rangle$  by assumption A.2.

$\langle 1, 0, 0 \rangle \succ \langle 0, 2, 0 \rangle$  by assumption A.1.

The result follows by transitivity. □

### Lemma

$$\langle 1, 0, 0, 0, 0, 0 \rangle \succ \langle 0, 0, 1, 1, 1, 1 \rangle$$

### Proof.

$\langle 0, 0, 2, 0, 2, 0 \rangle \succ \langle 0, 0, 1, 1, 1, 1 \rangle$  by assumption A.2

$\langle 0, 1, 0, 1, 0, 0 \rangle \succ \langle 0, 0, 2, 0, 2, 0 \rangle$  by assumption A.1

$\langle 0, 1, 1, 0, 0, 0 \rangle \succ \langle 0, 1, 0, 1, 0, 0 \rangle$  by assumptions A.2 and T.1

$\langle 1, 0, 0, 0, 0, 0 \rangle \succ \langle 0, 1, 1, 0, 0, 0 \rangle$  by the previous Lemma □

# Adam's first induction proof

- 1 is  $(1, 0, \dots, 0)$  preferred to no apples for  $k$  days, and a single apple on each of the next  $2^k$  days, and then none forever after?
- 2 previous lemmata have shown this to be true for  $k = 1$  and  $k = 2$
- 3 **assume** it to be true for some  $k = n$  (the **inductive hypothesis**)
- 4 if being true at  $k = n$  makes it true at  $k = n + 1$ , it is true for all  $k$



# Adam applies induction to apples

- 1 **assume** that  $\langle 1 \rangle \succ \underbrace{\langle 0, \dots, 0 \rangle}_n \underbrace{\langle 1, \dots, 1 \rangle}_{2^n}$
- 2 if  $\underbrace{\langle 0, \dots, 0 \rangle}_n \underbrace{\langle 1, \dots, 1 \rangle}_{2^n} \succ \underbrace{\langle 0, \dots, 0 \rangle}_{n+1} \underbrace{\langle 1, \dots, 1 \rangle}_{2^{n+1}}$ , transitivity gets the result
- 3  $\underbrace{\langle 0, \dots, 0 \rangle}_{n+1} \underbrace{\langle 2, 0, \dots, 2, 0 \rangle}_{2^n} \succ \underbrace{\langle 0, \dots, 0 \rangle}_{n+1} \underbrace{\langle 1, \dots, 1 \rangle}_{2^{n+1}}$  by A.2
- 4  $\underbrace{\langle 0, \dots, 0 \rangle}_n \underbrace{\langle 1, 0, \dots, 1, 0, 0 \rangle}_{2^n} \succ \underbrace{\langle 0, \dots, 0 \rangle}_n \underbrace{\langle 0, 2, \dots, 0, 2, 0 \rangle}_{2^n}$  by A.1
- 5 if we can show that  $\underbrace{\langle 1, \dots, 1 \rangle}_{2^n} \succ \underbrace{\langle 1, 0, \dots, 1, 0 \rangle}_{2^n}$ , we're done
- 6 equivalent to  $\underbrace{\langle 1, \dots, 1 \rangle}_{2^{n-1}} \succ \underbrace{\langle 0, 1, \dots, 0, 1 \rangle}_{2^{n-1}}$ , which holds by repeated use of A.1 and T.1
- 7 finally, as  $\mathbf{a}^*$  lasts  $T < 17 + 2^{17}$  days,  $\langle 1 \rangle \succ \mathbf{a}^*$

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# Bad notation

How could the notation below be improved?

*Consider two companies “A” (the protection buyer) and “B” (the protection seller) who agree on the following. Consider a protection time window  $(T_a, T_b]$ , meaning that protection will be negotiated for defaults happening between times  $T_a$  and  $T_b$ . (Brigo, Morini, and Pallvicini, 2013, p.54)*



## Uniqueness results

From the FT's 2013 Quiz of the Year:

*Below are synopses of the highest-grossing films in 2013 in three different territories. Can you name each country? And which of the three films is the highest-grossing in its country's 100-year film-making history?*

- a) A 16th-century holy man tries to become a demon hunter by tackling a fish demon and a pig demon and going on a quest to find the Monkey King. In one scene, he saves himself from falling by sticking his fingers in another person's nostrils.*
- b) The grandfather of the protagonist dies aged 99 while watching a cricket match. Our hero agrees to take his grandfather's ashes to a distant village but his train trip is interrupted by the arrival of a woman being chased by four armed men: she's trying to escape from a forced marriage. High jinks ensue.*
- c) A playboy-industrialist challenges a vicious terrorist to take him on. The terrorist does so. Meanwhile a biotech entrepreneur has invented a treatment that regenerates missing limbs, with unfortunate side effects. Mayhem ensues.*