Class 1: Net Present Values

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Grace and John Tarera slaughter goats for a living. Demand is brisk: Malawians adore goat stew. The Tareras want to expand their business to meet this demand, but they lack capital. Mrs Tarera thinks they need about 20,000 kwacha ($250). This may not sound much, but in Malawi, where the average annual income is only about $200, it can take years to raise such a sum.[1]

1. Alile Zimba, a friend of the Tareras, hears about the success of their goat slaughtering business and considers starting one of her own. To make the payment of 20,000 kwacha required now to establish the business she would need to borrow the money. If she does start it, she will earn 12,000 both this year and next.[2] The opportunity cost of capital (the rate charged by local money lenders) is 40% per annum.

   (a) What is the net present value of Alile's project?

   (b) Should be project be undertaken?

   (c) What is the highest hurdle (or interest) rate at which the project is worth undertaking?

   (d) What is the NPV at that rate?

[1] "Want to make the poor less poor? Give them proper title to what they own", *The Economist*, 29 March 2001.

[2] Assume that years are single points in time.

(e) Suppose that Alile had 20,000 kwacha of personal savings, and therefore does not need to borrow the money. How does this change the analysis?

2. Suppose that Alile and her brother, Nashon, have no collateral that would be recognised in a court; no one will lend them money. Thus, their savings are the only source of funding for the goat project. Fortunately, they each have 10,000 kwacha so that, collectively, they can fund the goat slaughtering project (if they do, they will split the proceeds equally). Nashon is a bit of a lad, so that he only cares about the present. Alile, on the other hand, cares equally about this year and next year.

   (a) would Alile want to undertake it?

   (b) would Nashon want to undertake the project?

   (c) how could Nashon and Alile take advantage of access to capital markets? What is the value of their doing so?

3. Consider a *perpetuity* that pays an amount C every year into the infinite future. Assuming a constant discount rate of r and no risk of default:

   (a) write an algebraic expression for V_∞ the NPV of the perpetuity; and

   (b) reduce the infinite summation derived above to express V_∞ as a single term. (Hint: re-write the infinite summation to find an occurrence of V_∞ on the right hand side; then solve for V_∞.)

4. Consider an *annuity* that pays an amount C every year for T years. Making the same assumptions as in question 3:

   (a) write an algebraic expression for V_T the NPV of the annuity;

   (b) using the technique hinted at in question 3b, reduce the summation to a simpler expression; and

   (c) re-derive the expression in part 4b by *replicating* the annuity: buying a perpetuity now for V_∞, and then selling it in period T (for $\frac{1}{(1+r)^T}V_\infty$, in present value terms).