This question is based on Tirole (2005, Exercise 5.1).

Consider a version of the fixed investment model of moral hazard developed in Tirole (2005, §3.2) in which an entrepreneur with no cash holdings \((A = 0)\) has an investment opportunity this period, and another one next period, each of which requires \(I\) to finance; assume that there is no discounting. The projects are identical: each yields \(R > 0\) with probability \(p\) and zero otherwise; each succeeds with probability \(p_H\) if the entrepreneur exerts effort, and \(p_L < p_H\) otherwise, with \(\Delta p \equiv p_H - p_L\); for each project, the entrepreneur obtains private benefits of \(B\) by shirking, and zero otherwise. The entrepreneur and investors are all risk-neutral; the latter operate in a competitive market with zero interest rate.

1. If \(p_H R > I > p_L R + B\) for each project, then is NPV maximised by funding neither, one or both of the projects (assuming that their incentive compatibility constraints can be satisfied)?

2. Consider a long-term contract that begins by funding the first project. If it succeeds, the contract certainly funds the second project; if the first project fails, the second is only funded with probability \(\xi \in (0,1)\). If the entrepreneur is paid \(R_b\) if and only if both projects succeed:

   (a) if the first project succeeds, what is the entrepreneur’s incentive compatibility constraint in the second period?
(b) if the first project fails, what is the entrepreneur’s incentive compatibility constraint in the second period?

(c) what is the entrepreneur’s incentive compatibility constraint in the first period?

(d) which of the three values derived above will be written as $R_b$ into the contract, and why?

(e) what are the investor’s expected costs?

(f) what is the investor’s expected revenue? (Hint: it may help to draw a decision tree, and list the expected revenue under each outcome.)

(g) what, therefore, is the investor’s participation constraint? (Hint: when $\xi = 0$ it reduces to the expression in Tirole (2005, Exercise 5.1.i).)

3. Suppose that the first project fails.

(a) Why are there incentives to renegotiate the long-term contract described in problem 2?

(b) What $\xi'$ (the new probability of funding the second project) and $R_b'$ (the incentive compatible return to the entrepreneur in the event of the second project’s success) would the entrepreneur and investor renegotiate to after a first failure?

(c) If the entrepreneur expects to renegotiate to $\xi'$ and $R_b'$ after an initial failure, what incentive compatibility constraint does she actually face in the first period? Solve for $R_b''$, her incentive compatible revenue in the event of two successes.

(d) Compare the magnitudes of $R_b'$ and $R_b''$ and provide an intuition for the difference.

References