

Class 4: long-term contracts

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This question is based on [Tirole \(2005, Exercise 5.1\)](#).

Consider a version of the fixed investment model of moral hazard developed in [Tirole \(2005, §3.2\)](#) in which an entrepreneur with no cash holdings ($A = 0$) has an investment opportunity this period, and another one next period, each of which requires I to finance; assume that there is no discounting. The projects are identical: each yields $R > 0$ with probability p and zero otherwise; each succeeds with probability p_H if the entrepreneur exerts effort, and $p_L < p_H$ otherwise, with $\Delta p \equiv p_H - p_L$; for each project, the entrepreneur obtains private benefits of B by shirking, and zero otherwise. The entrepreneur and investors are all risk-neutral; the latter operate in a competitive market with zero interest rate.

1. If $p_H R > I > p_L R + B$ for each project, then is NPV maximised by funding neither, one or both of the projects (assuming that their incentive compatibility constraints can be satisfied)?
2. Consider a long-term contract that begins by funding the first project. If it succeeds, the contract certainly funds the second project; if the first project fails, the second is only funded with probability $\xi \in (0, 1)$. If the entrepreneur is paid R_b if and only if both projects succeed:
 - (a) if the first project succeeds, what is the entrepreneur's incentive compatibility constraint in the second period?

- (b) if the first project fails, what is the entrepreneur's incentive compatibility constraint in the second period?
 - (c) what is the entrepreneur's incentive compatibility constraint in the first period?
 - (d) which of the three values derived above will be written as R_b into the contract, and why?
 - (e) what are the investor's expected costs?
 - (f) what is the investor's expected revenue? (Hint: it may help to draw a decision tree, and list the expected revenue under each outcome.)
 - (g) what, therefore, is the investor's participation constraint? (Hint: when $\xi = 0$ it reduces to the expression in [Tirole \(2005, Exercise 5.1.i\)](#).)
3. Suppose that the first project fails.
- (a) Why are there incentives to renegotiate the long-term contract described in problem 2?
 - (b) What ξ' (the new probability of funding the second project) and R'_b (the incentive compatible return to the entrepreneur in the event of the second project's success) would the entrepreneur and investor renegotiate to after a first failure?
 - (c) If the entrepreneur expects to renegotiate to ξ' and R'_b after an initial failure, what incentive compatibility constraint does she actually face in the first period? Solve for R''_b , her incentive compatible revenue in the event of two successes.
 - (d) Compare the magnitudes of R'_b and R''_b and provide an intuition for the difference.

References

- J. Tirole. *The Theory of Corporate Finance*. Princeton University Press, 2005.