PARETO-IMPROVING INCOME TAX REFORM

October 2004

Peter Sinclair*

University of Birmingham, Edgbaston, Birmingham, B15 2TT, UK

*: email – p.j.n.sinclair@bham.ac.uk; telephone: 0121.414.6742

Preliminary: please do not quote without the author's permission.

1: Introduction

This paper is not strictly about *optimal* income tax arrangements. Instead, it is concerned with *improving* a hypothetical tax system, and in particular, seeing whether certain types of reform can bring gain to everyone. This is probably the stiffest test any reform could meet. The hypothetical tax system against which alternatives will be compared is a linear one, where everyone faces a common rate of tax and receives an identical transfer, whatever their circumstances. This is a particular example of a "flat tax".

The main alternative that this paper considers is a "quadratic" tax system. Here the proportion of transfer-adjusted original income that the agent is allowed to retain – the "retention ratio" - is a linear, decreasing function of the transfer. This makes your consumption a quadratic function of the benefit you claim. The idea is that there is a particular transfer (and therefore tax rate) that will maximize the disposable income of someone with given original income. Those with little will go for a big transfer and a higher tax rate; those with more, for less of both[1]. In a sense, this system creates a market in tax rates. The fiscal authority sells marginal rate cuts, at a schedule of prices; everyone chooses how much, if any, to buy; and what they pay is deducted from (and might even exceed) the transfer they receive. We shall start, in section 2, by exploring this idea in detail, and concentrating upon Rawlsian optimal tax-benefit functions, within the context of a rectangular distribution of earning abilities and a common, symmetric Cobb Douglas utility function. Later sections broaden the approach in various ways.

[1] This is a simple, concrete way of applying the concept of self-selection espoused, inter alios, by Weymark (1987) and Brito et al (1990).

As the model explored in this paper has just a single period, and abstracts from capital and intertemporal choices, the type of flat tax explored here is a cut-down, static version of the proposal adumbrated by Hall and Rabushka (1995, 1996), and a major focus of discussion in Atkinson (1996). There is one important difference, however: the flat tax proposal usually excludes a specific transfer, replacing it with an *allowance* of initial income that is tax-exempt. If the allowance is A, and the flat rate on all income above A is t, everyone above this threshold gets an implicit transfer of tA. But those with no original income get nothing. In what is below called the linear income tax system, we shall, essentially, be examining a fiscal rule that gives this transfer to *everyone*, while, at the same time, subjecting it to tax. So the net of tax transfer given to everyone would be $t(1\text{-}t)A$.

Much of the literature on the flat tax, and the contrast it bears to existing tax systems, concentrates on the question of whether a switch would alter the long run growth rate in an endogenous growth setting, and, if so, how much. Devereux and Love (1994), Judd (1998) and Stokey and Rebelo (1995) are important contributions devoted to this issue. A recent analysis by Cassou and Lansing (2004) – which unlike several previous studies finds, with qualifications, that linearizing the income tax schedule *is* growth-promoting – provides a useful, short, up to date survey.

By contrast, the focus of the present paper is on not on the dynamic aggregate effects, but on the distributional consequences, and the essential comparison here is not between a flat tax and any actual system currently in place, typically one where marginal rates rise (unevenly) with income, but rather between a flat tax and some alternatives to it where marginal tax rates actually *decline* with income. While the distributional effects of moving from status quo arrangements *towards* a flat tax are considered in detail by Atkinson, and briefly (for income deciles) by Seldon and Boyd (1996), among others, the comparisons studied in this paper have not been drawn before.

There is a large literature on the shape of optimal non-linear income taxes, which begins with Mirrlees (1971). Unfortunately the main *general* result is that the marginal rate of tax faced by the highest tax payer should be zero, if his or her utility is to count positively towards social welfare. Below this point, general conclusions are unavailable. Kanbur and Tuomala (1994) support the arguments of Seade (1977, 1982) and find, for particular cases, that it could be inverted U-shaped, highest at intermediate incomes; but Diamond (1998) presents a case where utility is linear in consumption and concave in leisure[2], which can make it U-shaped. For Weymark (1987), where utility is again quasi-linear but concave in consumption, individuals self-select the tax and transfer system that suits them best, and a single-crossing property gives this a firm basis for inferring optimal tax rates, but little can be said about the shape of the optimal tax function; and Brito et al (1990), while showing how single-crossing can be relaxed, concur. Although Brito et al emphasize the fact that the Mirrleesian optimum income tax rates decline only slightly, and do not therefore deviate that greatly from a flat tax, perhaps one of the very few safe general statements is that a linear income tax regime is almost certain to be dominated by a non-linear one. This proposition is the starting point for the investigations in the present paper.

Progressivity relates primarily to whether the *average* rate of tax rises with income. On this definition progressivity throughout the full range of incomes is quite compatible with a declining *marginal* rate, so long as the marginal always exceeds the average. Possibly the simplest example that demonstrates this compatibility is a system where the total tax collected from an agent with income *y* consists of the sum of two elements, a positive one which is a fraction of *y*, and a negative one (a transfer) which is inversely proportional to *y*: $\alpha y - \beta / y$. (Here, α and β are both positive, and α <1). The average rate of tax rises with income, with derivative $2\alpha y^{-3}$. But the marginal rate declines, with an equal and opposite derivative. This example shows that progressive tax systems do not have to tax income at an increasing marginal rate.

[2] Roemer et al (2003) also work with this assumption.

The structure of the paper is as follows. The basic case is examined in section 2. Section 3 extends this to consider other welfare functions, while section 4 relaxes the assumption of a uniform ability distribution. Generalizations of the quadratic form for taxation are discussed in section 5. Section 6 takes a look at two other utility functions, and section 7 concludes.

2: Analysis of the Basic Case

Transfer payments are essential when recipients have very meagre opportunities for income. Without them, their marginal utilities of consumption would be very high. This points to the need for transfers from those better off, at least when social welfare is influenced by Benthamite considerations. And when social welfare is deemed to be tilted towards the poorest, that conclusion emerges with still greater force. But if transfers are paid to the better off as well, both national income, and the tax receipts that will help to finance the transfers, are liable to fall appreciably. This conclusion is certain if, as seems indubitable, leisure is a normal good. There is therefore an important case for concentrating transfers on those in greatest need. Payments to the poor will also have this effect, but the loss of tax receipts, following any induced reduction in their work, will be much more modest than for those with greater earning opportunities. For high earners, transfers may in fact be negative – promoting extra work by these individuals – and it seems natural that some consideration should be given to lowering their marginal income tax rate as a quid pro quo for this.

One way of achieving this concentration is to means-test the benefits, clawing them back as income rises. This paper will study a particular form of clawback. The idea to be explored here is that everyone is presented with a menu of tax-benefit arrangements. You can opt for a high positive transfer, at the cost of a higher tax rate, or, alternatively, for a low – possibly negative – transfer and a lower marginal tax rate. Those with low earning opportunities will opt for the former combination, and those more fortunate will go for the latter.

A simple example of this menu could be the linear scheme

$$t(w) = \xi + \theta b(w) \qquad (1)$$

Here, $t(w)$ denotes the income tax rate that will be chosen by an individual facing a wage rate *w*, and $b(w)$ is the transfer that will be selected to accompany it. The parameters ξ and θ are taken to be positive, and faced by all. We shall assume that original income consists solely of wage earnings (wh, where *h* denotes hours of work) and that income tax is levied on $wh + b$. The tax-benefit system implied by (1) can be called *the quadratic income tax system,* or QIT for short. What makes it quadratic is the fact that all agents' disposable incomes are parabolic functions of the transfers that they can choose. So the natural idea is that each should pick the transfer that maximizes disposable income.

This implies that the agent faces two problems. First, he or she needs to choose the level of the transfer, *b(w)*, that maximizes net disposable income (or equivalently consumption, in a one period setting). Second, labour supply *h* is selected to maximize utility, subject to the previous choice of tax-benefit regime. The first of these choices is independent of the utility function, while the second clearly isn't. The first problem, Max $[1 - \xi - \theta b(w)][wh + b(w)]$, implies

$$b(w) = [-wh + (1 - \xi)/\theta]/2 \qquad (2)$$

and the second involves choosing *h* to maximize

$$U = U(\theta[wh + (1 - \xi)/\theta]^2 / 4, 1 - h) \qquad (3)$$

where *1-h* denotes leisure. There is a side constraint that requires labour to be non-negative. In (3), the first argument of the utility function is *b*-optimized consumption, implied by (2).

For the rest of this paper, we shall focus on the symmetric Cobb-Douglas utility function, $U = \ell nc + \ell n(1-h)$ where c denotes consumption. This is the function examined by Mirrlees (1971) in his classic paper. The solution for labour implied by maximizing this form of (3) with respect to *h* is

$$h(w) = Max[0, 2(w - w^*)/3] \quad (4)$$

where *w** is the "critical wage", that is the highest wage rate associated with a zero labour supply; scrutiny reveals that $w^* = (1-\xi)/2\theta$. Only those agents whose earning ability exceeds *w** will participate in the labour force. The level of consumption associated with (4) will be

$$c(w) = \theta\{Max[w^{*2}, (w + 2w^*)^2/9]\} \quad (5).$$

The next stage is to derive the relationship between the tax parameter θ and the critical wage implied by an aggregate resource constraint. If government spending on output, the single good produced, is *G*, labour is the sole factor of production, and $\psi(w)$ is the distribution of wage rates, total demand in the economy (*D*, call it) will be found by integrating (5) across abilities:

$$D = G + \int_0^{w^*} \psi(w)\theta w^{*2} dw + \int_{w^*}^{\infty} \psi(w)\theta[\{w + 2w^*\}^2/9]dw \quad (6)$$

and the corresponding expression for total supply, *S*, will be

$$S = \int_{w^*}^{\infty} \psi(w) 2\{[w - w^*]/3\} dw \qquad (7)$$

Equating (6) and (7) gives an explicit solution for θ in terms of *w** and *G*, which depends upon the properties of the ability distribution. The equality of (6) and (7) is tantamount to assuming that the government balances its budget.

The parameter θ tells us how sharply the tax rate increases as the transfer increases. The other fiscal parameter, ξ, is the tax rate selected by anyone who opts for a zero transfer. Since we have seen that $\xi = 1 - 2\theta w^*$, revealing that this tax rate declines as ability rises, its role in the function $\theta(w^*, G)$ has already been incorporated. Precisely where the authorities decide to set θ will depend upon its behaviour and objectives. At one extreme, it may determine this parameter to maximize the utility of the least advantaged (the Rawlsian principle). The best known alternative maximand is the Benthamite conception of social welfare as the (arithmetic) mean of all citizens' utilities. Other possibilities, reflecting democratic pressures and political competition, would at least include a role for median utility (the "marginal voter" in a binary-choice fiscal referendum), tempered perhaps by the effects of party political contributions by the richest. And Roemer's recent work (1999, and, with others, 2003) on an "equal opportunities" welfare approach may perhaps be interpreted as a modified form of Rawlsianism[3]. The simplest and most general statement might be to write social welfare, *W* call it, as some function

$$\phi(w^*, \theta, X) = \phi(w^*, f(w^*, G), X) \qquad (8)$$

where *X* denotes a vector of other potentially relevant considerations, including direct government spending *G*, and possibly also issues connected to envy (Nishimura, 2003) or poverty as a public bad (Wane, 2001).

---

[3] Other possibilities include the "equal sacrifice principle" suggested, and probed, by Mitra and Ok (1997).

A benevolent government choosing *w** optimally, with *G* given, would fix both *w** and θ at an interior equilibrium, if there is one, where $\phi_1 + f'\phi_2$ vanishes and $\phi_{11} + 2\phi_{12} f' + \phi_{22}[f']^2 + \phi_2 f''$ is negative. But buried inside these conditions are complex issues related to the properties of the utility and ability distribution functions, as well as the way ϕ aggregates different agents' utilities. The clearest results emerge from combining a rectangular distribution of abilities, on the interval (0,1) with our Cobb Douglas utility function and Rawlsian social welfare.

Under these conditions, the Rawlsian optimum critical wage is related to *G* by

$$2w^* = [13 - 12G]^{0.5} - 3 \qquad (9)$$

so that *w** is at its highest, $\sqrt{3.25} - 1.5 \approx .302$, when *G* is zero, and vanishes if *G* attains its maximum feasible value of one third. By contrast, if θ is set to zero, and the authorities adopt a *linear income tax system* (LIT), with a uniform tax rate *t*, a common transfer *b*, and a critical wage rate u, minimum utility will equal

$$\ell n(1-t) + \ell nu = \ell n[(1-u)^2 - 4G] \quad - 2\ell n(1+u) + \ell nu \qquad (10).$$

(10) is maximized where $u = \sqrt{5-4G} - 2$. The value of *u* is again at its highest when government has no direct expenditure, and vanishes when *G* is at its ceiling of one quarter. The constant rate of income tax starts at 61.8% when *G* is zero, and climbs to reach 100% when G equals one quarter.

The key question from the standpoint of this paper is how the QIT and LIT systems compare in their effects on the distribution of utilities. The answer is that *everyone is better off* when the tax system is quadratic rather than linear. The shift from LIT to QIT generates a Pareto improvement.

This can be demonstrated in three steps. First, contrast the utilities of those who work under neither system. Under QIT, the utilities of such people will each equal

$$\ell n\theta + 2\ell nw^* = \ell n[(1-w^*)^2 - 3G] + 2\ell n3 - 3\ell n(1+2w^*) + 2\ell nw^* = \ell n\frac{9(x-2)}{8} + 3\ell n\frac{x-3}{x-1}$$

while under LIT, they will each be

$$\ell n(1-t) + \ell nu = \ell n\frac{u\{(1-u)^2 - 4G\}}{(1+u)^2} = \ell n\frac{2(y-2)^2}{y-1}.$$

Here, $x \equiv \sqrt{13-12G}$ and $y \equiv \sqrt{5-4G}$. Scrutiny reveals that QIT generates higher utility for these individuals than LIT, no matter what the value of G throughout its feasible range ($0 \le G \le 1/4$).

Next, consider the utility of someone with w=w* who works under LIT but not under QIT. With LIT, on the other hand, utility equals

$$\ell n(1-t) + 2\ell n[(w^*+u)/2] - \ell nw^*.$$

This reduces to $2\ell n\frac{(y-2)(x-7+2y)}{2(y-1)} - \ell nw^*$. Again, QIT delivers higher utility for this person than LIT. It is easiest to see this by first setting G to zero, then by seeing that the utility gap rises as G increases.

The third step is to consider people who participate in the labour force under both systems. We begin by taking the utility difference, $\Delta(w)$ call it, for someone with a wage rate w. This will be

$$\Delta(w) = \ell n\theta - \ell n(1-t) - 2\ell n3/2 + 3\ell n(w+2w^*) - 2\ell n(w+u) \tag{11}$$

The value of w at which (11) is minimized, $\tilde{w}$ say, is $\tilde{w} = 4w^* - 3u$. Substituting this into (11) reveals that $\Delta(\tilde{w})$ is positive when *G*=0, and furthermore that $\Delta(\tilde{w})$ increases unambiguously when *G* goes up. This is seen most easily by first fixing *G=0*, and showing that $\Delta(\tilde{w})$ is positive here, and then differentiating $\Delta(\tilde{w})$ with respect to *G* to reveal a positive derivative. . With (11) positive at $w = \tilde{w}$, it must be positive, *a fortiori*, at other wage rates in the region. Together, these findings imply that the entire distribution of utilities is higher under QIT than LIT. Consequently, QIT Pareto-dominates LIT.

It is also worth seeing what QIT implies for the fiscal parameters, θ and ξ. Take the case when *G* vanishes, so the entire proceeds of income tax go to paying transfers. In this case *w** equals $\sqrt{3.25} - 1.5$, or nearly 0.303. This equals $(1-\xi)/2\theta$. The value of θ at which the government's budget balances will be 1.0571. So ξ will equal .3599. These numbers imply that the transfer for each member of the poorest, non-working, 30.3% of the population will equal .303 (pre-tax), and that the income tax rate they face will be almost 68%. By contrast, at the top of the ability distribution, the income tax rate is just over 43.4%, and the (taxable) transfer they receive is positive but a good deal smaller (0.0704). In after tax values, the transfer to each of the poorest is 0.0969, nearly two and a half times as large as the transfer at the top (0.398).

Under LIT, with no direct government spending, everyone receives a transfer of .236 before tax, and 0.0902 after tax. By contrast, one major feature of QIT is that transfers to the better off fall, and go up for the worse off. A consequence of this is that, although the less able work a little less, the abler work appreciably longer, generating higher national income. Original labour income goes up, for someone at the top of the distribution with a unit wage, from under 0.382 to nearly 0.465. Despite the lower tax rate, the effect of the extra work (and lower transfer) ensures that the top earner contributes 11% more to the fiscal authority under QIT than LIT (.16204, as against 0.1459). This is true not just for those at the top: almost a quarter of the ability distribution pays more tax, net of transfers. And it is these extra payments that are able to finance more generous transfers lower down.

The LIT system is hampered by the constraint that the same transfer is paid to all. For those at the bottom of the distribution, this transfer is very valuable, because it is really all they have to live on. But higher up the scale, the transfers are both less needed (because their recipients' labour incomes are higher) and more damaging (in terms of valuable labour reduced). Where the QIT system scores is by introducing extra degrees of freedom, that can be exploited to trim the transfers to the better off; and the tax rate cuts are needed to persuade them to accept the transfer cuts. Put another way, the QIT sets up a kind of market where individuals can purchase reductions in the rate of tax they face.

3: Other welfare functions

If the Rawlsian LIT is Pareto-dominated by the Rawlsian QIT, does the same hold when the Benthamite QIT is contrasted with the Benthamite LIT? This is the question to which we now turn.

The answer is no, it does not. There is a small group (whose wage rate is close to $4w^*$-$3u$) who prefer the LIT to the QIT, when w^* and u are chosen to maximize arithmetic mean utility. Everyone else gains from the switch to QIT, and mean utility goes up, but there is not a Pareto improvement. This is true, at least, when government's direct spending on goods is zero. But if G is positive, a Pareto improvement may occur. The condition for this is that G should exceed about 0.11. The intuition is that the tax rate needs to be large enough, for reductions higher up to bring *universal* benefit. The Benthamite marginal tax rate is liable to be quite low unless government spending needs are large. The Benthamite social welfare

Deleted:

function can also generate a Pareto improvement, even with no government spending, if it is defined not as a simple utility sum, but weighted (exponentially, for example) with bigger weights lower down: this requires a sufficiently high degree of social aversion to inequalities in utility.

The Rawlsian and Benthamite welfare functions can be blended. The simplest example is the Roberts Social Welfare Function, proposed by Roberts (1980)[4], which is an arithmetic average of the two. Here, a Pareto improvement arises from a switch from a Roberts-optimal LIT to a Roberts-optimal QIT so long as the weight on minimum utility is no less than about one half, when G is zero. With positive government spending, the minimum weight on minimum utility drops, and keeps dropping as G rises.

Another possible welfare function blends minimum and average utility with *median* utility. The notion is that the government may consider holding a referendum on the tax-benefit system, and temper its benevolence with some concern for its outcome. Or there may be political duopoly and elections, which might, in certain cases, elevate the median voter to the status of dictator. A special case of this is a welfare function that is simply the arithmetic weighted sum of minimum and median utility. In this case also, the switch from optimal LIT to optimal QIT will generate a Pareto improvement if the weight on minimum utility is large enough.

4: Other Ability Distributions

Another special feature of the basic case considered is the assumption that earning abilities are uniform on the interval (0,1). How does varying the distribution affect the results? This is the question to which we turn in the present section.

[4] P 430.

Unfortunately there are not many transparent general findings to report. We shall explore them, and glean what we can, but greater illumination tends to come from experimenting with various alternatives that permit integration of relevant terms, and establishing the conditions, if any, under which switching from LIT to QIT leads to a Pareto improvement.

What can be said in general? Within the confines of the assumption about preferences (that they are symmetric, Cobb-Douglas between leisure and consumption, with all original income earned by labour), about social welfare (that it is Rawlsian), and about government spending (that it vanishes), the LIT displays some properties that are invariant to modifications of the ability distribution. If w_L^*, u_L are defined as the Rawls-optimal critical wage and unemployment rates respectively, and $\psi(w)$ is the ability distribution, the value of w_L^* is related to u_L by

$$\int_{w_L^*}^{\infty} (w - w_L^*)\psi(w)dw = w_L^* \sqrt{2(1-u)}\,;$$

this implies that $1-t = [1+\sqrt{\frac{2}{1-u_L}}]^{-1}$, which does not vary at all with the properties of $\psi(w)$. This is not true, however, of the key fiscal parameter in the QIT system, θ. The Rawls-optimal value of θ is related to w_Q^*, u_Q (the Rawls-optimal values of the critical wage and unemployment rates respectively) by

$$\theta = \frac{2[\int_{w_Q^*}^{\infty} (w - w_Q^*)\psi(w)dw]/3}{w_Q^{*\,2} u_Q + \{\int_{w_Q^*}^{\infty} [2w_Q^* + w]^2 \psi(w)dw\}/9} \equiv \frac{A}{B} = \frac{3(A - w_Q^*(1-u_Q))}{w_Q^*[A + w_Q^*(2+u_Q)]}\,.$$

The numerator of this expression, *A*, is aggregate income, and the denominator, *B*, represents the ratio of aggregate consumption to θ.

The person who gains least from the switch from LIT to QIT, with ability $w^{**} = 4w_Q^* - 3w_L^*$, receives a utility gain of

$$\Delta^{**} = \ell n\theta - \ell n(1-t) + \ell n(2w_Q^* - w_L^*)$$

which reduces to

$$\ell n3(2 - \frac{w_L^*}{w_Q^*})(1 + \sqrt{\frac{2}{1-u_L}})(1 - \frac{w_Q^*(7+2u_Q)}{2A + 3w_Q^*(2+u_Q)}).$$

This expression is positive for certain if the excess of w_Q^* over w_L^* is small, and must at first increase locally as the gap rises, but can go negative if the gap is pronounced when the unemployment gap is not particularly sensitive to the critical wage gap. This can arise when the ability distribution displays positive skew.

Retaining the Rawlsian basic case, let us now examine particular modifications of the distribution of abilities. Three transformations are explored:

(a) a spread-preserving shift in mean earning ability;

(b) a third moment shift, by replacing rectangularity by a negative exponential distribution;

(c) a mean-preserving shift in both second and fourth moments, by replacing rectangularity with isosceles triangularity.

The first of these makes $\psi(w)$ uniform on $(\varepsilon, 1+\varepsilon)$ rather than on (0,1). The second defines $\psi(w) \equiv \sigma\exp(-\sigma w)$, and the third retains the distribution's unit base and mean, while imposing a triangle whose other sides are both $\sqrt{17}/2$.

Variation (a) reinforces everyone's rise in utility, right across the distribution, that comes about when the Rawlsian QIT replaces the Rawlsian LIT. Variation (c) also exhibits an unambiguous Pareto improvement when this change is introduced. These results hold when government spending is zero, and hold *a fortiori* when it is positive.

Variation (b) is, however, troublesome. When G is zero, not everyone gains. Indeed everyone with a wage rate above about 0.395 is worse off, and that large majority is independent of the parameter σ. A Pareto improvement is still possible, but requires a sufficiently large ratio of G to σ. In other words, the bigger the positive skew in the ability distribution, the higher government spending must be for the Rawls-optimal QIT to deliver a Pareto improvement over the Rawls-optimal LIT.

There are cases, however, when positive skew does not destroy Pareto improvement, even when G vanishes. The truncated distribution $\psi(w) = 1.5\ w^{-0.5}, 0 \le w \le 1$, so that the unit area is $\int_0^1 (1.5/\sqrt{w})dw$, is a case in point. The individual who gains least from the change from LIT to QIT, who commands a wage rate of 0.4444, just benefits from it. It is the elimination of the right tail that achieves this.

Another example is the distribution

$$\psi(w) = \frac{1}{2(e-2)} Min[(e^w - 1), (e-1)e^{-\lambda w}] \qquad (12)$$

where $\lambda = (e-1)/(e-2)$ and $0 \le w \le \infty$. The untruncated distribution (12), which peaks at w = 1, and converges to zero at its two limits, also generates a Pareto improvement when a Rawls-optimal QIT replaces a Rawls-optimal LIT, even when G vanishes. The second of these two cases, (12), is particularly interesting, since its right tail is left in place, the terms in $w\psi(w)$ and $w^2\psi(w)$ are analytically integrable, and its general shape bears a reasonably close similarity to the lognormal ability distribution considered by Mirrlees (1971). Distribution (12) illustrates the fact that a positive skew does not *have* to prevent a Pareto improvement resulting from a switch from LIT to QIT.

5: Generalizing the Quadratic Income Tax System

The QIT is not the only alternative to the linear income tax system. The retention ratio R might be written

$$R(b(w)) = (1-\xi - \theta b(w))^{m} .$$

We could call this the *Polynomial Income Tax System* (PIT). The PIT reduces to the quadratic in the special case where m=1. But one can entertain the possibility of *any* positive value of m. With m given, w uniform on (0,1) and u chosen to maximize minimum utility, a switch from LIT to PIT generates a Pareto improvement, with G zero, so long as m does not exceed about 6.6. It is worth noting, however, that if m, and not just u, is set to maximize minimum utility, m will rise to infinity, destroying the Pareto improvement result. A second type of generalization is

$$R(b(w)) = e^{\alpha(1-\xi-\theta b(w))} .$$

This might be called the *Exponential Income Tax System* (EIT). Under the same conditions – no government spending, a uniform ability distribution with lower support at zero, and a Rawlsian choice of $w*$, the wage rate above which workers supply positive labour - the EIT generates some remarkable features. The first of these is that pre-tax, post transfer incomes, $wh+b(w)$, are equalized. This strongly egalitarian property is balanced, however, by an even greater surprise: the effective marginal tax rate at top incomes (for those with w=1) is in fact zero. That property is a well known general feature of optimum income tax under Benthamite social welfare, but it is rather startling to see it emerge here in a Rawlsian optimum, too.

The QIT, and its variants, allow individuals to select the benefit and marginal tax rate that suits them best. But it is worth asking whether non-linearizing the tax schedule can still benefit everyone even if the transfer is uniform. Consider an isoelastic, increasing transformation of the retention ratio, *R:*

$$R = \eta(wh + b)^{-\lambda}$$

so that an individual's consumption is related to her labour income by

$$c = \eta(wh + b)^{1-\lambda} .$$

This generalizes the LIT, to which it tends as $\lambda \to 0$ and $\eta \to 1 - t$. Marginal income tax rates are declining (increasing) with income if λ <(>)0. But everyone gets the same transfer, pretax. We shall continue to work with a uniform distribution of wage rates, with supports at 1 and 0, and set government spending at zero; and we shall concentrate on the Rawlsian version of this system. If the parameters *w** and λ are set to maximize minimum utility, $\eta b^{1-\lambda} = \eta[w*(1-\lambda)]^{1-\lambda}$, subject to the resource constraint that aggregate income $(\{(1-\lambda)(1-w^*)^2 / 2(2-\lambda)\})$ balances aggregate consumption $(\eta\{(1\text{-}\lambda)[1 + w*(1-\lambda)]/(2-\lambda)\}^{2-\lambda} /(1-\lambda))$, we find that $\lambda \approx -0.6135, w^* \approx 0.245459$ and $\eta \approx 0.418177$. Consumption of anyone who does not work is nearly 0.94, as against 0.09016 with LIT. So those in this position are better off than under LIT. But importantly, so is everyone else, including the individual who gains least from the switch (whose wage rate is about 0.4586). His utility gain is just above 0.056. So a Pareto improvement arises here, too. The effective marginal rate of income tax on an infinitesimal increase in labour income is 45.25% at the top (*w* =1) and about 76.3% at the bottom (*w**=.245459).

This last finding is important, since it shows that a Pareto improvement can result simply from making marginal tax rates decline with income – the QIT feature that transfers fall for the better off does not apply here. In fact, since the pretax transfers are uniform, and the retention rates rise with income (λ<0), the better off actually receive slightly larger explicit post-tax transfers than the worse off. On the other hand, the fact that the marginal tax rates are declining means that there is an *implicit* reduction in the transfer for the better off, because intramarginal income is taxed more heavily than marginal income.

A final variant of the QIT to be considered here allows QIT and LIT to apply at different income levels. Suppose everyone is given a choice between the two systems, in the simplest setting (abilities uniform on (0,1), no government spending, Rawlsian welfare). The difference in their utilities will be

$$U_Q - U_L = \ell n(4\theta / 27(1-t)) + 3\ell n(w + \frac{1-\xi}{\theta}) - 2\ell n(w+u) .$$

So long as $w > 2\frac{1-\xi}{\theta} - 3u$, this difference increases with *w*. So higher earners prefer the QIT. Let *w*** denote the wage rate of someone indifferent between the two systems. From the above, this is the solution to

$$[w^{**} + \frac{1-\xi}{\theta}]^3 = 3(3(w^{**}+u)/2)^2(1-t)/\theta .$$

The condition that income balances consumption in the aggregate implies

$$\theta = 9L / 4(1+\frac{1-\xi}{\theta})^3$$

where $L \equiv 3(w^{**}-u)^2 + 4(1-w^{**2}) - 4(1-w^{**})\frac{1-\xi}{\theta}$. Combining these two conditions and recognizing that minimum utility equals *u(1-t)*, we may write the Rawlsian maximand as

$$\frac{uL}{3(w^{**}+u)^2}[\frac{\theta w^{**}+1-\xi}{\theta+1-\xi}]^3 .$$

If the parameters w^{**}, u and $(1-\xi)/\theta$ are set to maximize this, we find that $(1-\xi)/\theta = 3u - w^{**}$, and $8(w^{**}-u) = 1+17u-\sqrt{-15+66u+288u^2}$, so that $u \approx .323, w^{**} \approx .3806$ and $(1-\xi)/\theta \approx .5884$.

So the poorest 8.5% of those in work, as well as those out of work, opt for LIT, and the rest go for QIT. The individual who gains least from the switch from pure LIT to the QIT/LIT mix has a wage rate of 0.4686. Calculation reveals that he or she gains. So the Rawls-optimal QIT/LIT mix Pareto dominates the pure LIT. In the former, the linear marginal tax rate for those lower down the distribution is 69.84%, a little higher than under pure LIT, with a more-than-compensating gain from the higher (constant) benefit, *b*. The marginal tax rate declines for those with wage rates above 0.38, reaching its minimum of 19.64% at the top. This example also reveals that the QIT/LIT mix is superior to the pure QIT from the standpoint of the poorest, and therefore that it is better – in the circumstances examined, at least – for marginal tax rates *not* to decline with rising incomes towards the bottom.

In these examples, and indeed throughout the paper, we have found it useful to work not with tax rates, but with retention ratios – the fractions of original income, plus net transfers selected, that individuals can retain and consume. The QIT makes the retention ratio linear, and of course declining, in the size of the transfer. The PIT and the NIT are just transforms of that retention ratio.

6. Other utility functions

Up to now we have worked with the symmetric Cobb Douglas utility function, in its additively separable form, $U = \ell nc + \ell n(1-h)$. Rewriting it by, say, $U = [c(1-h)]^a$ (with a >0) will make no difference to the pure Rawlsian cases, since these maximize any increasing function of the consumption of the poorest who will not be working. But it will affect the Benthamite analysis, since the way mean utility behaves is altered.

This short section looks briefly at two other utility functions, to provide some insurance against the possibility that the Pareto-improvement resulting from a switch from LIT to QIT could be a unique feature of the Cobb Douglas. These functions are:

(a) $U = \ell nc - h$

and (b) $U = -c^{-1} - h$.

Both are quasi-linear, displaying a constant marginal disutility of work. We shall continue to operate under the simplest assumptions: Rawlsian welfare, no *G*, and wage rates uniform on (0,1).

With (a), LIT gives a uniform tax rate of about 0.5437, and an unemployment rate of 29.55978%. Under QIT, *u* rises to 43.9087%, and $\theta =$ 0.872029. Repeating our earlier approach, we find that the person that gains least from the change from LIT to QIT has a wage rate of 0.5825762, and experiences a positive net ga%in in utility of just over 0.03. So a Pareto improvement ensues.

With (b), LIT gives a tax rate of 63.188%, and unemployment of 13.9628%, approximately. Under QIT, unemployment rises to 30.03558%, and $\theta = 0.252439$. This too generates a Pareto improvement, with the biggest gains close to the mean of abilities (almost 0.14 at *w*=0.5) and lowest, but still positive, at the top (not quite 0.053 at *w*=1). A Pareto improvement occurs in both of these two cases. So we can be sure that this consequence, under the stated assumptions, of the move from the flat to the quadratic tax – benefit system is not confined to Cobb Douglas preferences.

7: Conclusions and Qualifications

If one is prepared to take a particular definition of aggregate welfare, tax reforms that are capable of increasing it are not particularly hard to find, even when we insist that individuals' labour choices and incentive compatibility conditions are fully respected. But it is asking far more of a tax reform that *everyone's* welfare should go up. This is the challenging hurdle to which we have submitted a particular alternative to the flat income tax, with its uniform exemption limit (or rather, in our case here, uniform transfer) and constant marginal rate. The main rival against which the flat tax has been pitted is a quadratic tax-benefit system, in which individuals trade off the marginal tax rate they face against the size of the transfer they request.

Those capable of earning only meagre labour income will opt for a big positive transfer and a high marginal rate of tax, levied on the sum of the transfer and their original income. Those with higher earning ability will choose less of both, and supply more labour in consequence – not just more labour than their less fortunate peers, but a good deal more labour, typically, than they would have chosen with the flat tax. It is this increase in the labour supply on the part of the abler members of society that generates an increase in aggregate output, and, with it, the opportunity for government to enlarge the transfers for the very poor. Not just this: it turns out, under a wide class of conditions, that utilities go up right across the board. The switch from the flat tax to the quadratic brings a Pareto improvement. For the able, the extra work means less leisure, but the utility effect of this is more than offset by the private benefits of their extra consumption.

This result is shown to occur in a basic case where social welfare is Rawlsian, and ability uniform. When government spending is high enough, Pareto improvement is also shown to occur when social welfare is Benthamite (and the tax parameters are set in accordance with that maximand). The uniform distribution is sufficient for these results, but not necessary: it has been shown to be robust in the face of various modifications (but not all of those we have examined).

A final generalization relates to the quadratic character of the tax-benefit system. This too can be generalized, without removing the Pareto improvement finding. In one particular generalization, an implicit top tax rate of zero is found to characterize a Rawlsian optimum. This is a well known attribute of optimal tax systems when the utility of the highest earner is included in social welfare, first discovered by Mirrlees (1971) and generalized by Seade (1982). It is quite a surprise to see it featuring in a particular Rawlsian optimum as well.

These conclusions do not imply that a flat tax is inferior to tax and benefit systems currently in operation. Those systems usually exhibit increasing marginal tax rates[5], at least at higher income levels. So removing this feature, which a flat tax would entail, might well represent gain. But the logic of this paper is that such a reform, though possibly welcome from various standpoints, would not in fact be going far enough, at least in the context of the examples explored here. Better still to have marginal tax rates that *decline* with income, at least over a higher range, and accompany them with transfer payments that are clawed back quite sharply as income rises. Better still not just from abstract concept of social welfare: better still, within the confines of the model examined above, for *everyone.*

Nonetheless, the confines of that model merit emphasis again: preferences have been symmetric Cobb Douglas between leisure and consumption (except in section 6), and common to all; the analysis has been limited to a single period; social welfare has never deviated from some (weighted) average of mean, median and minimum utility, and mostly consisted solely of the last of these; and nearly all the analysis has focused on particular (and particularly convenient) ability distributions. Lastly, it must be stressed again that this paper is not attempting to derive general propositions about optimal tax schedules, and restricts itself, rather, to the humbler task of contrasting particular ones.

[5] It applies in all the (OECD) income tax systems examined by Roemer et al (2003) and Wagstaff et al (1999).

One puzzle that this paper pinpoints is the issue of why income tax systems in practice do display increasing marginal rates, if, at least in the setting considered here, everyone can be shown to benefit so often by making them decrease in income, instead. There could be a number of explanations for this. Part of the reason may be that means-testing benefits in fact makes actual, effective marginal rates decline with income over quite a broad range of lower and even middle incomes. Another possibility is that the worse off are envious of the better off[6], and prepared to sacrifice something to see the gaps reduced. Further, it might be that preferences and ability distributions accord more closely with those studied by Diamond (1998), which reveal that a U shaped marginal tax structure could be best after all. And if income were at least *thought* to be exogenous, stripping the optimum income tax issue of all the fascinating complexity that stems from the principle that the size of the cake depends on how it is to be shared, political competition might well lead to an increasing marginal rate[7]. Voting might also lead to overtaxation – and perhaps more at the top end – if the skills of the unemployed are unknown[8]. Finally, an uncharitable observer could also argue that part of the reason may be that some politicians lack a firm grasp of the distinction between marginal and average rates of tax, and think – mistakenly – that progressivity requires marginal rates to rise with income.

References

Atkinson, A.B. (1996), *The Basic Income / Flat Tax Proposal*, Lindahl Lectures on Monetary and Fiscal Policy, Oxford: Oxford University Press

Brito, D.L., J.H.Hamilton, S.M. Slutsky and J.E. Stiglitz (1990), Pareto Efficient Tax Structures, *Oxford Economic Papers*, 42, 61-77

Carbonell-Nicolau, O. and E.F. Klor (2003), Representative Democracy and Marginal Rate Progressive Income Taxation, *Journal of Public Economics*, 87, 1137-1164

[6] Nishimura (2003) finds that considerations of envy could raise optimal marginal income tax rates, given that leisure is a luxury.

[7] Carbonell-Nicolau and Klor find that this occurs in some Nash equilibria, and in any strong Nash equilibria. But in exogenizing income, they deprive the income tax problem of much of its interest. Roberts (1977) was the first to consider a voting-over-income tax model, but his marginal tax rates are linear.

[8] Laslier et al (2003) find this.

Cassou, S.P and K.J. Lansing (2004), Growth Effects of Shifting from a Graduated-Rate Tax System to a Flat Tax, *Economic Inquiry*, 42, 194-213

Devereux, M. B. and D.R.F. Love (1994), The Effects of Factor Taxation in a Two Sector Model of Endogenous Growth, *Canadian Journal of Economics*, 27, 509-536

Diamond, P.A. (1998), Optimal Income Taxation: an Example with a U Shaped Pattern of Optimal Marginal Tax Rates, *American Economic Review*, 88, 83-95

Hall, R.E. and A. Rabushka (1995), *The Flat Tax*, 2nd edition, Stanford, Hoover Institution Press

Hall, R.E. and A. Rabushka (1996), The Flat Tax: a Simple Progressive Consumption Tax, in M. Boskin, ed., *Frontiers of Tax Reform*, Stanford, Hoover Institution Press

Judd, K.L. (1998), Taxes, Uncertainty and Human Capital, *American Economic Review Papers and Proceedings,* 88, 289-292

Laslier, J.F., A. Trannoy and K. van der Staeten (2003), Voting Under Ignorance of the Job Skills of the Unemployed: an Overtaxation Bias, *Journal of Public Economics*, 87, 595-626

Kanbur, R. and M. Tuomala (1996), Inherent Inequality and the Optimal Gradation of Marginal Tax Rates, *Scandinavian Journal of Economics*, 94, 275-282

Mirrlees, J.A. (1971), An Exploration in the Theory of Optimum Income Taxation, *Review of Economic Studies*, 38, 175-208

Mitra, T. and E.A. Ok (1997), On the Equitability of Progressive Income Taxation, *Journal of Economic Theory*, 73, 316-334

Nishimura, Y. (2003), Optimal Non-linear Income Taxation with Reduction for Envy, Journal of Public Economics, 86, 363-386

Roberts, K.W.S. (1977), Voting over Income Tax Schedules, *Journal of Public Economics*, 8, 329-340

Roberts, K.W.S. (1980), Interpersonal Comparability and Social Choice Theory, *Review of Economic Studies,* 47, 421-439

Roemer, J.E et al. (2003), To What Extent Do Fiscal Regimes Equalize Opportunities for Income Acquisition among Citizens?, *Journal of Public Economics*, 87, 539-565

Sadka, E. (1976), On Income Distribution, Incentive Effects and Optimal Income Taxation, *Review of Economic Studies*, 43, 261-268

Seade, J. (1977), On the Shape of Optimal Tax Schedules, *Journal of Public Economics*, 7, 203-235

Seade, J. (1982), On the Sign of the Optimum Marginal Income Tax, *Review of Economic Studies*, 49, 637-644

Seldon, B.J. and R.G. Boyd (1996), *The Economic Effects of a Flat Tax*, NCPA Policy Report 205, Dallas: National Centre for Policy Analysis

Stokey, N. L. and S. Rebelo (1995), Growth Effects of Flat Rate Taxes, *Journal of Political Economy,* 103, 519-550

Wagstaff, A. et al. (1999), Redistributive Effect, Progressivity and Differential Tax Treatment: Personal Income Taxes in Twelve OECD Countries, *Journal of Public Economics*, 72, 73-98

Wane, W. (2001), The Optimal Income Tax when Poverty is a Public 'Bad', *Journal of Public Economics*, 82, 271-299

Weymark, J.A. (1987), Comparative Static Properties of Optimal Nonlinear Income Taxes, *Econometrica*, 55, 1165-1185

Deleted: